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Three essays on behavioral economics and mechanism design

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To the Graduate Council:

I am submitting herewith a dissertation written by Na Zuo entitled "Three essays on behavioral economics and mechanism design." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Economics.

Scott M. Gilpatric, Major Professor

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(Original signatures are on file with official student records.)

Three essays on behavioral economics and mechanism design

A Dissertation Presented for the

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Degree

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Dedication

To my mom, Xiaoxi Li

my father, Wenjian Zuo

For their advice, their faith, and their inspiration.

To my husband, Dong Yan

For all the things great and small.

To my little boy, Xu Yan

For all the happiness and love.

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Abstract

My three essays on behavioral economics and mechanism design introduce two new microeconomic theoretical models.

In the first chapter, we develop an n-player theoretical model applying the concept of Virtual Bargaining to study cooperative behavior in public goods games characterizing team production. Virtual Bargaining is a modeling framework that characterizes how players may construct a tacit agreement to coordinate behavior in the absence of explicit communication. Players identify their worst-possible payoff outcome from any candidate agreement, and mutually best-respond with respect to maximization of their worst-payoff function. Players face uncertainties regarding whether other players will follow through on a candidate agreement or play their Nash best response to candidate agreement. The worst payoff function is the minimum over these possibilities. We show that, relative to the Nash equilibrium predictions, the virtual bargaining model predicts more effective coordination with higher contributions to the public good when individual contributions are strategically complementary and the public good production technology exhibits decreasing return to scale. This type of public good characterizes team production processes when each player receives a share of the total team output, and effort by each player increases the marginal product of every other player.

In the second chapter, based on the different theoretical predictions of the VB and standard Nash models, we propose an experimental design to test whether the VB or the standard Nash model predicts people's behavior better. What's more, we also test the VB theory in a price-setting duopoly market environment.

In the third chapter, we show that hidden information regarding the quality of placements of online advertisements by publishers can lead to inefficiency in the market that is a form of

moral hazard. We then characterize an incentive-compatible contract between the publisher and a traditional contract advertiser in the online display advertisements environment. This solves the inefficient impression allocation problem between the traditional contract market and the real-time bidding spot market. Unlike previous papers, which rely on including the reputation or long-term benefit consideration into the publisher's objective function, our incentive-compatible contract characterizes a one-time profit maximization problem for the publisher.

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Chapter 1

Can Virtual Bargaining Explain Coordination in Public Good Games Characterizing Team Production?

1.1 Introduction

Many economic interactions are characterized as voluntary contributions to a public good. These are settings, ranging from effort as part of a team in a workplace to charitable giving, in which individual agents choose how much to invest in an endeavor which mostly benefits others because the benefits are shared among a group of two or more. The standard non-cooperative game theory yields a Nash equilibrium prediction of under-investment in public goods as each agent has an incentive to "free ride" on the contributions of others. This prediction obviously has significant predictive power—–free riding is a major problem in public good provision—but in fact, there is extensive evidence that individual agents contribute more than the Nash prediction in many settings. This evidence includes not only field settings such as charitable contributions but also controlled economics laboratory experiments. This gap between the standard economic theory and empirical observations has motivated researchers to seek theories to explain the phenomenon, such as other-regarding preferences. In this paper, we use the recently developed concept of virtual bargaining to explain coordination on supra-NE contributions in a specific type of VCM game. We consider games characterizing team production, where each individual's contribution is complementary to that of others, and all share equally in the value of the team-output produced.

The concept of virtual bargaining was initially introduced by Misyak and Chater (2014), explaining social interactions requiring coordination behavior across a range of activities. They argued that "the complex behavioral patterns—or 'unwritten rules'—governing such coordination emerge from an ongoing process of 'virtual bargaining'." In other words, participants may coordinate without communication as if they were to explicitly bargain with each other, based on a common understanding of how bargaining would take place among rational optimizing individuals. The concept of Virtual Bargaining has been formalized in economics by Melkonyan et al. (2017), who model tacit collusion among firms in an oligopoly market. They find that Virtual Bargaining predicts successful coordination in the Bertrand game, but not in the Cournot game. This result arises because prices are strategic complements in Bertrand competition, whereas quantities are strategic substitutes in the Cournot competition. Voluntary contribution games characterizing teamwork are similar to Bertrand games if effort contributions are strategic complements, and players will be mutually better off if they coordinate to contribute more than the NE prediction. Using the theory of Virtual Bargaining (VB) equilibrium developed by Melkonyan et al. (2017), we find that VB equilibrium contribution levels are higher than the Nash prediction but remain lower than the socially optimal level.

In the literature on public goods games, many papers study other-regarding preferences and incentives for cooperation (e.g., Experimental: Andreoni (1995), Goeree et al. (2002), Houser and Kurzban (2002), Brandts and Schram (2001), Palfrey and Prisbrey (1997); Theoretical: Andreoni (1990), Fehr and Schmidt (1999), Bolton and Ockenfels (2000)). The other-regarding models that explain over-contribution in public goods games include altruism, warm-glow, reciprocity, and inequity-aversion. And of course, people consider noisy behavior due to confusion as well (Brandts and Schram (2001)). Others have considered how a social preference for being a conditional cooperator may support coordination (Keser and Van Winden (2000), Fischbacher et al. (2001), Fischbacher and Gachter (2010)). In this framework, people's propensity to cooperate is conditional on the fact that other people are cooperating. However, as Ledyard (1994) concludes, there is no final agreement on what is driving the behavior patterns in public goods games. Moreover, none of these papers studies the specific type of public goods game where people's strategies are strategic complements. This paper is distinguished from the previous literature by focusing on public goods games that characterize teamwork, in which contributions are strategic complements. The Virtual Bargaining theory has important implications in this setting. Lappalainen (2018a) experimentally compared people's behavior in a traditional public goods game where contributions exhibit strategic substitutability and modified public goods game where contributions exhibit strategic complementarity. However, in their model, they implemented quadratic private consumption function, in which the marginal return from private consumption becomes negative beyond a certain point, which impacts behavior. In this paper, we use the traditional linear private consumption function, which doesn't have that problem.

The fundamental idea of virtual bargaining is that people can directly implement the results of the virtual bargain without explicitly going through the bargaining process when the results of the bargain are clear to both parties. Crucially, the ability to arrive at a VB equilibrium does not depend on learning and/or reputation effects, and may predict behavior even in a one-shot game.

1.2 Virtual Bargaining

We will formalize the notion of Virtual Bargain equilibrium in the context of a voluntary contribution game below. Here, we first provide an overview of the concept and its application to some simple games. In the virtual bargaining theory, each player considers every possible agreement that could be formed among the players, which are defined as the set of candidate agreements. When evaluating a candidate agreement, each player considers two scenarios. The first scenario is when all the players follow through on the agreement. The second scenario is the worst possible payoff he could face when some or all other players best respond to the agreement, each maximizing his own payoff conditional on others following through the candidate agreement. In this setting, best responses may, of course, be characterized as taking advantage of the other players by free-riding or behaving opportunistically rather than cooperatively. When considering uncertainties players face regarding whether others will follow through or best respond to a candidate agreement, virtual bargaining theory assumes players are maximally ambiguity-averse to this uncertainty. This means that the scenario which yields a player the lowest payoff guides his action. For example, consider a 2-player game between player i and j. If player i follows an agreement, being best responded to by the other player j will generate payoff π_{iA} to player i. If player j follows the agreement, this will generate payoff π_{iB} to player i. Then, between payoffs π_{iA} and π_{iB}, player i will use $\min(\pi_{iA}, \pi_{iB})$ to guide his behavior. This process identifies players' "worst-payoff function" for any candidate agreement. This payoff modeling is consistent with the maximin model of ambiguity aversion from Schmeidler and Gilboa (2004). In the maximin model of ambiguity aversion, players are so pessimistic that players will always use the scenario, which generates the lowest payoff to guide their behavior when they do not know the probabilities of each possible scenario. In the VB theory, we use the same idea to guide players' behavior when they are facing uncertainty regarding the strategies of their opponents. [1]

Since each player's worst-payoff function guides his behavior, an equilibrium defines a feasible agreement when no player can improve his worst payoff by a unilateral change of strategy. It is possible to have multiple equilibria, which thus define a set of feasible agreements. In that case, equilibrium selection criteria may be applied to identify a virtual bargaining equilibrium.

The logic of virtual bargaining equilibrium can be illustrated via the following example. Consider the Traveler's Dilemma game Basu (1994). This game has been used by Melkonyan et al. (2017) to illustrate how virtual bargaining differs from Nash predictions as well. In the game, the two players simultaneously and independently choose an amount of money between \$1 and \$100 in whole dollar increments. Both of the players will receive a payoff

[1] One thing to note here is that VB assumes that when the player best responds, he won't do it in a spiteful way. In other words, when the player has multiple best responses, among all the best responses in the best response set, the player will choose the one that generates the highest payoff for his opponents. Such an assumption is common in many strategic games, like ultimatum games.

equal to the lower value of the two chosen by the two players. In addition, the player who chooses the lower sum will receive a \$2 transfer from the player who chooses the higher sum (i.e. a small punishment to the "greedier" player and transformed to the more "modest" player). When the two players choose the same sum value, they will receive this amount, and no transfer will occur. In this game, both players' best response is to choose an amount \$1 less than what they believe their opponent chooses.

The Nash equilibrium of the game entails both players choosing \$1 and receives \$1 as a payoff. Virtual bargaining can lead to an equilibrium that yields higher payoffs for both parties. Consider the candidate agreement of both playing \$100. When one of the players chooses the strategy of \$100, he has to consider the two scenarios: First, if his opponent follows the agreement and chooses \$100 as well; alternatively, the opponent best responds and chooses \$99. In the first scenario, both of the players will receive \$100. For the second scenario, the player will receive \$97, and his opponent will receive \$101. \$97 is the worst payoff from this candidate agreement and according to the virtual bargaining theory, the player will use this payoff to guide his behavior. Consequently, even though the opponent might best respond to the agreement, following the candidate agreement and playing \$100 yields a higher payoff than choosing the Nash equilibrium of \$1. In this case, no matter whether the opponent following through the agreement or best responding to the candidate agreement, the player will be better off by following the candidate agreement. After considering all possible candidate agreements, it turns out that the discussed case is the one that yields the highest worst payoff for the player. Similarly, the opponent goes through the same process of finding the best candidate agreement. Then, both players will end up with complete coordination in the game, both choosing \$100.

In this specific case, players reach an equilibrium of complete coordination. However, in other coordination games, virtual bargaining does not generate complete coordination. In many cases, partial coordination results are achieved by virtual bargaining. However, the intuition of why we see some coordination behavior in games where Nash predicts the absence of coordination is the same.

1.3 The Model of Public Goods Game

In this paper, we consider an N-player symmetric one-shot public good game. $i = 1, 2...n$ denotes the N identical players. Each player has an initial monetary endowment of y_i. Each player simultaneously and independently decides how much of his endowment to invest in the public good, σ_i, with the remainder allocated to private consumption, $y_i - \sigma_i$. Players' utility from private consumption is in linear form, $X_i = y_i - \sigma_i$. Here we consider a public good game with strategically complementary investments. Contributions to the public good in this context can be thought of as effort allocated to team production. Strategic complementarity implies that in these games, one participant's contribution of effort will increase the marginal benefit of contributing effort by other participants. In our model, the function that represents the technology for the production of the public good from individual contributions is as follows. $P_i = M\sigma_i^{\frac{\alpha}{n}}(\prod_{j\neq i}\sigma_j^{\frac{\alpha}{n}})$, where P_i denotes the common value for all players of the public good. The parameter M scales the public good. The parameter α represents the degree of returns to scale in the team production, and also the degree of complementarity of the contributions from players. We constrain the parameter α in the range of $(0, 1)$, so the public good has the property of decreasing return to scale. The $\frac{\alpha}{n}$ exponential term insures that the returns to scale level is neutral to the number of players n, as $\sum_{i=1}^{n}\frac{\alpha}{n} = \alpha$. Each player's total utility is additive in private consumption and the public good. So $u_i = X_i + P_i$. With this, we can write

$$u_i(\sigma_i, \sigma_{-i}) = y_i - \sigma_i + M\sigma_i^{\frac{\alpha}{n}}(\prod_{j\neq i}\sigma_j^{\frac{\alpha}{n}}) \tag{1.1}$$

1.4 Nash Equilibrium and Socially Optimal Contribution Benchmarks

Before characterizing the virtual bargaining behavior, we first identify the Nash equilibrium and the socially optimal contribution levels.

By maximizing the utility function, we find player i's Nash best response function given other players' strategies σ_{-i}, is

$$R_i(\sigma_{-i}) = (\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} (\prod_{j\neq i} \sigma_j)^{-\frac{\alpha}{\alpha-n}} \tag{1.2}$$

In a N-player simultaneous-move game, when the initial endowment y_i for each player is large enough to ensure an interior solution, we have the NE level of investments in public good equal to $\sigma^{NE} = (\frac{\alpha}{n})^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$. When the initial endowment for each player is smaller than $(\frac{\alpha}{n})^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$, then we have a corner solution as $\sigma^{NE} = y_i$.

By maximizing the total utility of the N players, we identify the socially optimal level of investment in public good equal to $\sigma^{SO} = \alpha^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$ when each player's endowment is greater or equal than this level. The socially optimal investment of public good is $\sigma^{SO} = y_i$ if y_i is smaller than $\alpha^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$.

Note that both the Nash equilibrium and socially optimal contribution levels are finite, and thus interior to the choice space for y_i sufficiently large when $\alpha < 1$ (in other words, when the technology exhibits decreasing return to scale). The socially optimal contribution levels are the entirety of endowments, and the Nash best response may also diverge to the upper bond when $\alpha > 1$. We will confine the further analysis to $\alpha < 1$.

1.5 The Virtual Bargaining Model

The worst payoff of a candidate agreement $(\sigma_i^A, \sigma_{-i}^A)$ for player i is defined as the minimum utility over two scenarios. One is when all other players $j \in -i$ follow through on the candidate agreement $(\sigma_i^A, \sigma_{-i}^A)$ and play $\sigma_j^A,\ \forall\ j \in -i$; The second scenario is when all other players $j,\ \forall\ j \in -i$, best respond to the candidate agreement, according to the best response function defined by Eq (1.2). The reason is as follows. We show in appendix A.1 and A.2 that only symmetric candidate agreements can be feasible agreements, where feasible agreements are defined as candidate agreements such that no player can increase his worst payoff by unilaterally deviating from the agreement. [2] Given that, we only focus on symmetric candidate agreements. Furthermore, we define the worst payoff function as

[2] We show in Appendix A.1 that as long as the Virtual Bargaining best response function is weakly increasing, no asymmetric candidate agreement can be feasible agreement. And the weakly increasing property of VB best response function is an intrinsic property of the VB model when strategies are strategically complementary (proven in Appendix A.2).

the lower envelope of the two scenarios where either all other players follow through the candidate agreement or all other players best respond. Below is the structure of the proof that defining the worst payoff function as the minimum of the two cases (all other players follow through on the candidate agreement, or all other players best respond) is without loss of generosity.

First, we prove that Virtual Bargaining Best Response (VBBR) function is (weakly) monotonically increasing in other players' candidate agreement strategies even if the candidate agreement is asymmetric. Then we prove that the monotonicity of VBBR implies that only symmetric candidate agreements can be feasible. Therefore, the relevant set of candidate agreements is the set of symmetric candidate agreements. Conditional on symmetric candidate agreements, the worst-payoff arises from either all other players following the candidate agreement or all other players best responding. Therefore, the relevant worst-payoff function is the lower envelope of either all other players following the candidate agreement or all other players best responding.

We assume that in the worst-payoff calculation, each player may Nash best respond to a candidate agreement, but not iterate by best responding to others' best responses. The worst payoff function for player i is defined as follows,

$$w_i(\sigma_i^A, \sigma_j^A) = \min \left\{ \pi_i(\sigma_i^A, \sigma_j^A), \sup_{\sigma_j \in R_j(\sigma_{-j}^A)} \pi_i(\sigma_i^A, \sigma_j) \right\} \quad \forall j \in -i \tag{1.3}$$

where $\pi_i(\sigma_i^A, \sigma_j^A)$ represents the payoff of the candidate agreement (σ_i^A, σ_j^A) for player i with player i committing σ_i^A investment in the public good and the other players $j \in -i$ committing σ_j^A investment. The latter term represents the scenario when the opponents Nash best respond to the candidate agreement. $\sup_{\sigma_j \in R_j(\sigma_{-j}^A)} \pi_i(\sigma_i^A, \sigma_j)$ means that among all such Nash best response strategies for player j, $\forall j \in -i$, player j will choose the one that yields the highest payoff for other players $-j$. In other words, the VB model assumes players might deviate from the agreement, but they will do so only to maximize their own payoff, not do so in a spiteful way. In our model, since the players have a unique Nash best response, this assumption is not critical.

Given that players are focused on the "worst payoff" instead of "payoff", we define the set of candidate agreements such that no player can improve his worst payoff by a unilateral deviation as *feasible agreements*. The idea of *feasible agreements* is just like the Nash equilibrium in the world of "worst payoffs" instead of "payoffs". We demonstrate the concept of worst payoff and feasible agreements by the symmetric game of Table 1.1.

Table 1.1 (a) contains the normal form of a game where the action space has been reduced to three discrete alternatives. Table 1.1 (b) contains the associate worst payoffs for all pure-strategy profiles. In this game, each player $i = 1, 2$ has three pure strategies: H, M, and L. In the normal form of the game with standard payoffs, the Nash equilibrium is (L, L). In the Virtual Bargaining game defined over worst payoffs, there are two feasible agreements such that no player can improve his worst payoff by a unilateral deviation. One is (L, L), which coincides with the Nash equilibrium. The other one is (H, H), which generates a higher worst payoff for both players. To demonstrate that the latter profile (H, H) is feasible, one can demonstrate that any unilateral deviation from (H, H) will yield a player a lower worst-payoff. Thus (H, H) is a feasible agreement in addition to the profile (L, L).

Notice that with the strategically complementary property of this game, when players Nash best respond to a candidate agreement, they don't have the incentive to deviate to a far lower contribution. As a result of that, the deviating behavior will not generate a substantial negative impact on the payoff for the other player who follows through the candidate agreement. For example, given the candidate agreement profile (H, H), when one of the players best responds to the candidate agreement, it is his best response to deviate to strategy M but not the strategy L. Then the payoff to the player who follows through on the candidate agreement only decreases to 25 instead of 18 if the deviating player plays L.

To generalize the full game with a continuum of contributions in the choice space, we identify the worst payoff function based on the public goods game introduced in section 1.3. Based on the production technology of the public goods game, we first identify the payoff for player i for the scenario when all other players coordinate with the candidate agreement, which can be stated as

$$u_i(\sigma_i^A, \sigma_{-i}^A) = y_i - {\sigma_i}^A + M{\sigma_i}^{A\frac{\alpha}{n}}(\prod_{j\neq i}{\sigma_j}^{A\frac{\alpha}{n}}) \tag{1.4}$$

For the scenario when all players other than i best respond to the candidate agreement, we have to substitutes all other players' Nash best response function, $R_j(\sigma_{-j}^A)$, $j \in -i$, into player i's payoff function, $u_i(\sigma_i^A, \sigma_j^A)$. Then we have player i's payoff stated as

$$u_i(\sigma_i^A, \sigma_j^A) = y_i - {\sigma_i}^A + \alpha^{-\frac{\alpha(n-1)}{\alpha-1}} M^{-\frac{\alpha(n-2)-1}{\alpha-1}} {\sigma_i}^{A^{-\frac{\alpha^2(n-2)-\alpha}{\alpha-1}}}(\prod_{j\neq i}{\sigma_j}^A)^{-\frac{\alpha^2(n-2)}{\alpha-1}} \tag{1.5}$$

Based on the above definition of worst payoff function, player i's worst payoff function for a given candidate agreement (σ_i^A, σ_j^A) can be stated as

$$w_i(\sigma_i^A, \sigma_j^A) = \begin{cases} y_i - {\sigma_i}^A + M{\sigma_i}^{A\frac{\alpha}{n}}(\prod_{j\neq i}{\sigma_j}^{A\frac{\alpha}{n}}) & \\ \qquad \forall\ \sigma_i^A > M^{-\frac{n}{\alpha}}\alpha^{-\frac{n}{\alpha}}(\prod_{j\neq i}{\sigma_j}^A)^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}} & \\ y_i - {\sigma_i}^A + \frac{\alpha}{n}^{-\frac{\alpha(n-1)}{\alpha-n}} M^{-\frac{\alpha(n-2)+n}{\alpha-n}} {\sigma_i}^{A^{-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2}}}(\prod_{j\neq i}{\sigma_j}^A)^{-\frac{\alpha^2(n-2)}{\alpha n-n^2}} & \\ \qquad \forall\ {\sigma_i}^A \leq M^{-\frac{n}{\alpha}}\alpha^{-\frac{n}{\alpha}}(\prod_{j\neq i}{\sigma_j}^A)^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}} & \end{cases} \tag{1.6}$$

where $j \in -i$.

Eq(1.6) implies that when player i's public good investment is relatively high compared with the sum of other players $-i$' investments, the scenario of other players $-i$ following the candidate agreement will generate the lower payoff for player i than the other scenario. Moreover, when player i's public good investment is relatively low compared with other players $-i$' investment, the scenario of being best responded by all other players will generate the lower payoff. The intuition of this is straightforward. That is, the candidate agreement is relatively attractive when the opponents are investing relatively high endowment to the public good. So if the other players best respond to the candidate agreement, that will have a detrimental effect on player i's payoff. Conversely, when the other player's investment in the public good is relatively low under the candidate agreement, the candidate agreement itself becomes the less attractive scenario for player i, generating lower payoff than the scenario of being best responded by the opponents.

To make the calculation clear, we use Δ to replace the product of all other players' investment in the public good under that candidate agreement, $\Delta = \prod_{j \neq i} \sigma_j^A$. Then we have the worst payoff function as below.

$$w_i(\sigma_i^A, \Delta) = \begin{cases} y_i - {\sigma_i}^A + M {\sigma_i}^{A\frac{\alpha}{n}} \Delta^{\frac{\alpha}{n}} \\ \qquad\qquad\qquad \forall\ \sigma_i^A > (\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}} \\ y_i - {\sigma_i}^A + \frac{\alpha}{n}^{-\frac{\alpha(n-1)}{\alpha-n}} M^{-\frac{\alpha(n-2)+n}{\alpha-n}} {\sigma_i}^{A^{-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2}}} \Delta^{-\frac{\alpha^2(n-2)}{\alpha n-n^2}} \\ \qquad\qquad\qquad \forall\ {\sigma_i}^A \leq (\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}}) \end{cases} \tag{1.7}$$

In Figure 1.1, we graphically illustrate one example of the worst payoff function, when parameters $\alpha = 0.3$, $n = 2$, $M = 12$, $y_i = 20$ and $\Delta = 10$.

where the horizontal axis represents player i's investment level into public good in candidate agreements, and the vertical axis represents the worst payoff for player i. The solid curve defines player i's worst payoff function, given the summation of all other players' investment levels to a certain level.

Based on players' worst payoff function, we define the VB best response function very similarly to the Nash best response. The VB best response function is the player's strategies that maximize the player's worst payoff function, $w_i(\sigma_i^A, \sigma_{-i}^A)$, given a candidate agreement, $({\sigma_i}^A, {\sigma_{-i}}^A)$.

The VB best response function for player i is stated as [3]

[3]The derivation of the Virtual Bargaining best response function is in Appendix A.3.

$$
R_i^F(\Delta) = \begin{cases}
(\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} \Delta^{-\frac{\alpha}{\alpha-n}} & \\
& \forall \quad \Delta \leq \frac{\alpha}{n}^{-\frac{n-1}{\alpha-1}} M^{-\frac{n-1}{\alpha-1}} \\
(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}} & \\
& \forall \quad \frac{\alpha}{n}^{-\frac{n-1}{\alpha-1}} M^{-\frac{n-1}{\alpha-1}} < \Delta \\
& \quad \leq (-\frac{\alpha n-n^2}{\alpha^2(n-2)+\alpha n})^{\frac{\alpha(n-1)}{\alpha n-n}} \frac{\alpha}{n}^{\frac{(\alpha-n)(n-1)}{\alpha n-n}} M^{-\frac{n-1}{\alpha-1}} \\
(-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2})^{\frac{\alpha n-n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \frac{\alpha}{n}^{-\frac{\alpha n(n-1)}{\alpha^2(n-2)+2\alpha n-n^2}} M^{-\frac{\alpha n(n-2)+n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \Delta^{-\frac{\alpha^2(n-2)}{\alpha^2(n-2)+2\alpha n-n^2}} & \\
& \forall \quad \Delta > (-\frac{\alpha n-n^2}{\alpha^2(n-2)+\alpha n})^{\frac{\alpha(n-1)}{\alpha n-n}} \frac{\alpha}{n}^{\frac{(\alpha-n)(n-1)}{\alpha n-n}} M^{-\frac{n-1}{\alpha-1}}
\end{cases} \tag{1.8}
$$

It is convenient to write the Virtual Bargaining best response function as a function of the geometric mean of all other players' contributions instead of the product of all other players' contributions under candidate agreement. We replace the product of all other players' contributions, Δ, with ${\delta^A}^{n-1}$ where δ^A representing the geometric mean of all other players' contribution [4]. Then, the modified VB best response function as a function of the geometric mean of all other players' contributions can be stated as the following.

$$
R_i^F(\delta^A) = \begin{cases}
(\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} {\delta^A}^{-\frac{\alpha(n-1)}{\alpha-n}} & \\
& \forall \quad \delta^A \leq \frac{\alpha}{n}^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}} \\
(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} {\delta^A}^{-\frac{\alpha(n-1)-n}{\alpha}} & \\
& \forall \quad \frac{\alpha}{n}^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}} < \delta^A \\
& \quad \leq (-\frac{\alpha n-n^2}{\alpha^2(n-2)+\alpha n})^{\frac{\alpha}{\alpha n-n}} \frac{\alpha}{n}^{\frac{(\alpha-n)}{\alpha n-n}} M^{-\frac{1}{\alpha-1}} \\
(-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2})^{\frac{\alpha n-n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \frac{\alpha}{n}^{-\frac{\alpha n(n-1)}{\alpha^2(n-2)+2\alpha n-n^2}} M^{-\frac{\alpha n(n-2)+n^2}{\alpha^2(n-2)+2\alpha n-n^2}} {\delta^A}^{-\frac{\alpha^2(n-1)(n-2)}{\alpha^2(n-2)+2\alpha n-n^2}} & \\
& \forall \quad \delta^A > (-\frac{\alpha n-n^2}{\alpha^2(n-2)+\alpha n})^{\frac{\alpha}{\alpha n-n}} \frac{\alpha}{n}^{\frac{(\alpha-n)}{\alpha n-n}} M^{-\frac{1}{\alpha-1}}
\end{cases} \tag{1.9}
$$

where δ^A represents the geometric mean of the contributions of all other players under candidate agreement.

[4] Set $\delta^A = \Delta^{\frac{1}{n-1}}$, then we have $\Delta = {\delta^A}^{n-1}$

Of course, for any symmetric candidate agreement, δ^A will be the contribution level common to all. The symmetric Virtual Bargaining equilibrium can then be represented graphically. Writing this form of the VB best response function, we can represent the equilibria as the intersections of the VB best response function and the 45° line, where player i's VB best response function, $\delta_i^A(\delta^A)$, is plotted against δ^A, the common candidate agreement contribution level.

In Figure 1.2, we graphically illustrate one example of the VB best response function, when parameters $\alpha = 0.6$, $n = 3$, $M = 12$. The horizontal axis represents the common individual contribution level of all other players' investment into public good under candidate agreement, δ^A, and the vertical axis represents player i's VB best response given δ^A. The dashed line is the 45° line. The intersections of player's VB best response function and the 45° line are symmetric equilibria for this game.

Imposing symmetry among the n players, we identify two feasible agreements, identified as the intersections of the Virtual Bargaining best response function and the 45° line. The lower one equals to $(\frac{\alpha}{n})^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$, which coincides with the Nash equilibrium. The higher feasible agreement's contribution level equals to $(-\frac{\alpha^2(n-2)+\alpha n}{\alpha n - n^2})^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}} (\frac{\alpha}{n})^{-\frac{\alpha(n-1)}{(\alpha-1)(\alpha(n-2)+n)}} M^{-\frac{1}{\alpha-1}}$. The set of the equilibrium levels of investments in the public goods (there might be single or multiple equilibrium level) is defined as feasible agreements. If no player can increase his worst payoff w_i by unilaterally deviating from the equilibrium level of investment in the public good, then this is an equilibrium belonging to the feasible set of agreements.

Implied by the Virtual Bargaining best response function, we summarize players' directions of incentive to deviate when the contribution level is not equal to any of the two feasible agreement levels in the following lemma 1.1.

Lemma 1.1. *When considering (symmetric) candidate agreements where the contribution level is below the Virtual Bargaining equilibrium level, players have the incentive to deviate upward; When considering (symmetric) candidate agreements where contribution level is above the Virtual Bargaining equilibrium level, players have the incentive to deviate downward.*

Given multiple symmetric feasible agreements in the feasible agreement set, the appropriate selection criteria for identifying a virtual bargain equilibrium, in this case, is payoff dominance. A Virtual Bargaining Equilibrium (VBE) is defined as the equilibrium belonging to the feasible agreement that maximizes the players' worst payoff level. [5]

According to the payoff dominance mechanism, among the two feasible agreements,

$\sigma^* \in \left\{ \left(\frac{\alpha}{n}\right)^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}, \left(-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2}\right)^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}} \left(\frac{\alpha}{n}\right)^{-\frac{\alpha(n-1)}{(\alpha-1)(\alpha(n-2)+n)}} M^{-\frac{1}{\alpha-1}} \right\}$, the latter one is the VB equilibrium. We denote it as $\sigma^{VB} = \left(-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2}\right)^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}} \left(\frac{\alpha}{n}\right)^{-\frac{\alpha(n-1)}{(\alpha-1)(\alpha(n-2)+n)}} M^{-\frac{1}{\alpha-1}}$. [6]

The main results of the VB model are summarized in the proposition below.

Proposition 1.2. *With decreasing return to scale public goods technology $\alpha < 1$, and strategic complementarity among contributions, there are two symmetric feasible agreements, which are interior to the choice space when all players have sufficient income. The lower contribution feasible agreement coincides with the Nash Equilibrium contribution level, the higher contribution feasible agreement (Virtual Bargaining Equilibrium) is strictly greater than the NE contribution level but less than the socially optimal contribution level.*

The key idea is the following. With strategic complementarity among contributions, when best responding to a candidate agreement where the candidate agreement contribution level is relatively high, even the Nash best responding strategy (taking advantage of players who follow through the candidate agreement) is a relatively small downward deviation from the candidate agreement contribution level such that the payoff of players following through on the candidate agreement is not greatly reduced. Consequently, the worst-payoff determined by the scenario of being best responded for players who follow through on the candidate agreement is lowered but remains relatively higher than the Nash equilibrium level. Every player in the candidate agreement has the common knowledge of their worst-payoff if other

[5] Melkonyan et al. (2017) uses the Nash bargaining solution to select among contributions of possibly asymmetric feasible agreements. As we are an N-player symmetric game, then the feasible agreement that generates the highest payoff for a player will be generating the highest payoff for all players. So the Virtual Bargaining Equilibrium contribution level payoff dominate Nash Equilibrium.

[6] For detailed proof of the latter equilibrium contribution level is strictly greater than the first one, please see A.4.

players Nash best respond to the candidate agreement. Then given that even the worst-payoff of a cooperative candidate agreement is higher than the Nash equilibrium level, players can tacitly contribute a high level of investments. Thus, such kind of cooperative candidate agreement (where contribution level is higher than the Nash equilibrium level) can be a feasible agreement.

1.6 Comparative Statics

We now consider how the Virtual Bargaining equilibrium responds to changes in the teamwork game, and how this relates to changes in the Nash equilibrium and the socially optimal benchmarks.

Proposition 1.3. *The Nash equilibrium, VB equilibrium and the socially optimal level of contributions increase as the scalar M increases. The equilibrium teamwork outputs*[7] *in the three cases are increasing in the scalar M as well.* [8]

This is intuitive. As M gets larger, the total return from the public goods gets bigger while the marginal return from the private consumption remains constant. This gives people incentives to contribute more to public goods in all situations. In Figure 1.3, we show the above discussion when the parameters $\alpha = 0.6$ and $n = 3$. This pattern applies to the Nash equilibrium teamwork output and the VB equilibrium teamwork output as well. We show the relationship in 1.4 where parameters $\alpha = 0.6$ and $n = 3$.

Proposition 1.4. *The socially optimal level of contribution is independent of the number of players, n; The Nash equilibrium and VB equilibrium level of contributions are decreasing in the number of players, n. Similarly, the socially optimal teamwork output is independent of the number of players, n, while the Nash and VB equilibrium teamwork outputs are decreasing in the number of players, n.* [9]

[7] The teamwork output is defined as $M(\sigma^{NE/VB\frac{\alpha}{n}})^n$.

[8] Given $\sigma^{SO} = \alpha^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$, $\sigma^{NE} = (\frac{\alpha}{n})^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$ and $\sigma^{VB} = (-\frac{\alpha^2(n-2)+\alpha n}{\alpha n - n^2})^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}} (\frac{\alpha}{n})^{-\frac{\alpha(n-1)}{(\alpha-1)(\alpha(n-2)+n)}} M^{-\frac{1}{\alpha-1}}$, it is obvious to tell that Nash equilibrium, VB equilibrium and socially optimal level of contributions are increasing in M. So are the total outputs.

[9] For detailed proof, please refer Appendix A.5

Figure 1.5 shows the relationship between the number of players, n, and the Nash/VB equilibrium contribution level, when the parameters $\alpha = 0.6$ and $M = 8$. As we can tell from Figure 1.5, both Nash and VB equilibria contributions decrease as the number of players increases. The intuition is that given the return to scale level, α, as the number of players n increases, the marginal return from one player's individual contribution is decreasing as $\frac{\alpha}{n}$ gets smaller. This is the free-rider problem grows as the number of players grows. This impacts both the Nash equilibrium and VB equilibrium.

Figure 1.6 shows the relationship between the number of players, n, and the Nash/VB equilibrium teamwork outputs[10], where the parameter setup is also $\alpha = 0.6$ and $M = 8$. As for the Nash/VB equilibrium teamwork outputs, they are decreasing in the number of players, n, as well. In general, the VB mechanism predicts greater output than the Nash mechanism, but coordination will be dampened as the number of players gets too large. So, the VB equilibrium teamwork output starts high above the Nash equilibrium teamwork output, but the difference becomes smaller as the number of players gets larger.

1.7 Conclusion

This paper implemented the Virtual Bargaining concept into a teamwork environment, where players' contributions to team-output are strategic complements. According to the Virtual Bargaining model, a higher level of cooperation than predicted by non-cooperative game theory can be sustained even without explicit communication among players when the production technology of the teamwork is decreasing return to scale. The results are robust in a generalized N-player one-shot game. Due to the strategic complementarity of contributions, best response behavior will not hurt the payoff too greatly if a player follows through on the candidate agreement. In this case, the worst scenario of being best responded generates a lower payoff than fully cooperation but remains higher than the Nash predicted equilibrium level. Under these circumstances, partial coordination with contribution levels higher than the Nash prediction but smaller than the socially optimal level among players can be feasible.

[10] The teamwork output is defined as $M(\sigma^{NE/VB\frac{\alpha}{n}})^n$.

Table 1.1: A discrete numerical example of Virtual Bargaining

	Player 2		
Player 1	H	M	L
(a) Normal form of the game			
H	(30, 30)	(25, 33)	(18, 32)
M	(33, 25)	(28, 28)	(20, 29)
L	(32, 18)	(29, 20)	(22, 22)
(b) Worst payoffs for pure strategies			
H	(25, 25)	(25, 20)	(18, 22)
M	(20, 25)	(20, 20)	(20, 22)
L	(22, 18)	(22, 20)	(22, 22)

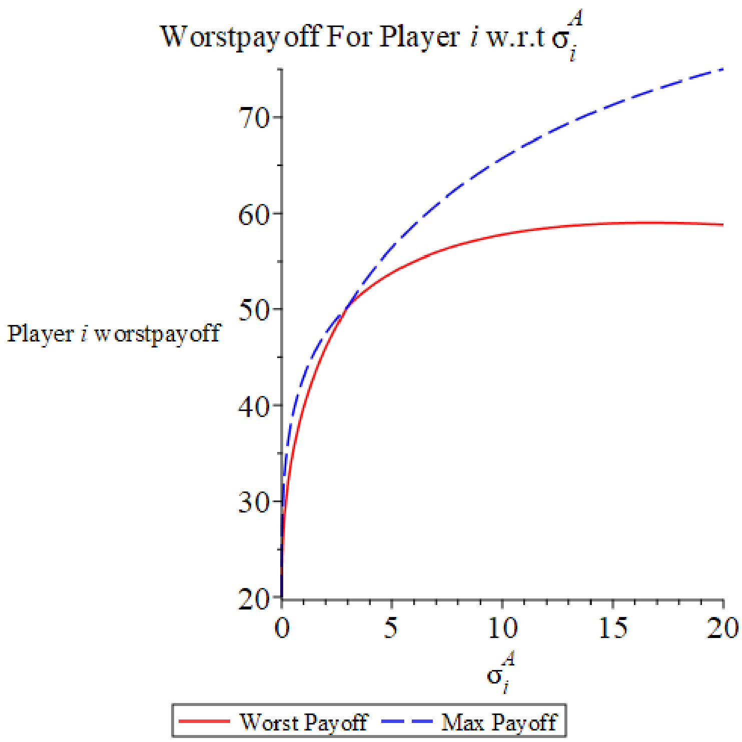

Figure 1.1: Worst-payoff function for player i

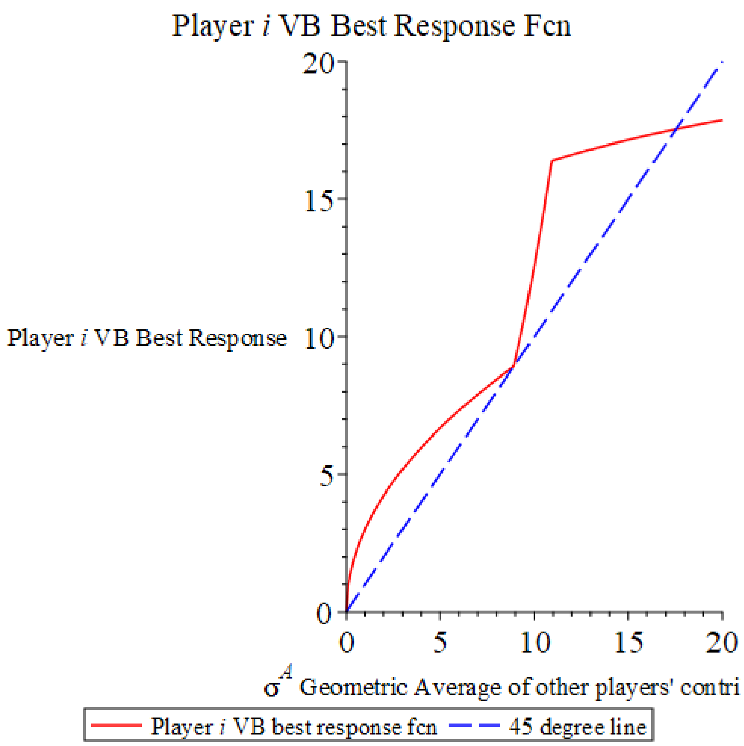

Figure 1.2: Virtual Bargaining best response function

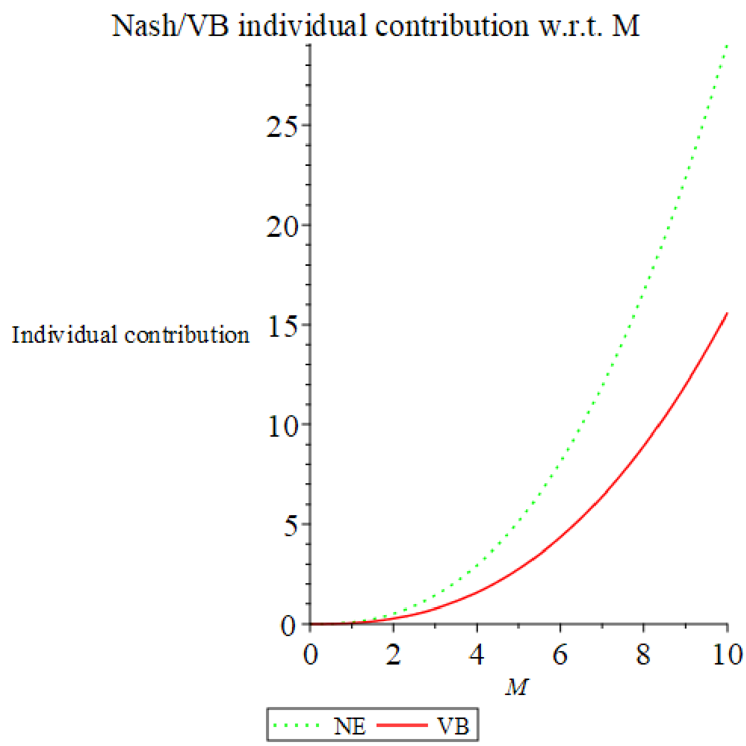

Figure 1.3: Nash and Virtual Bargaining contributions as M varies.

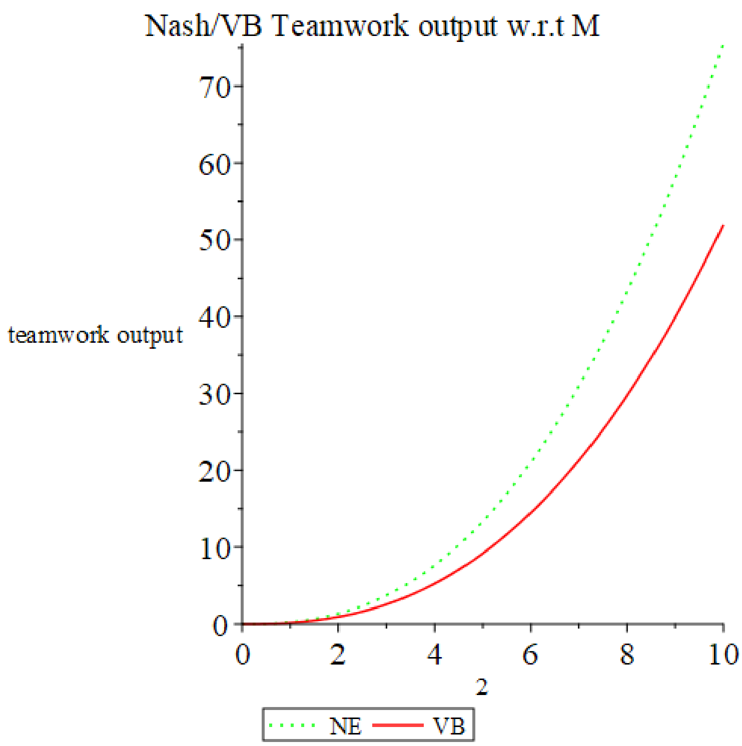

Figure 1.4: Total Nash and Virtual Bargaining contributions as M varies.

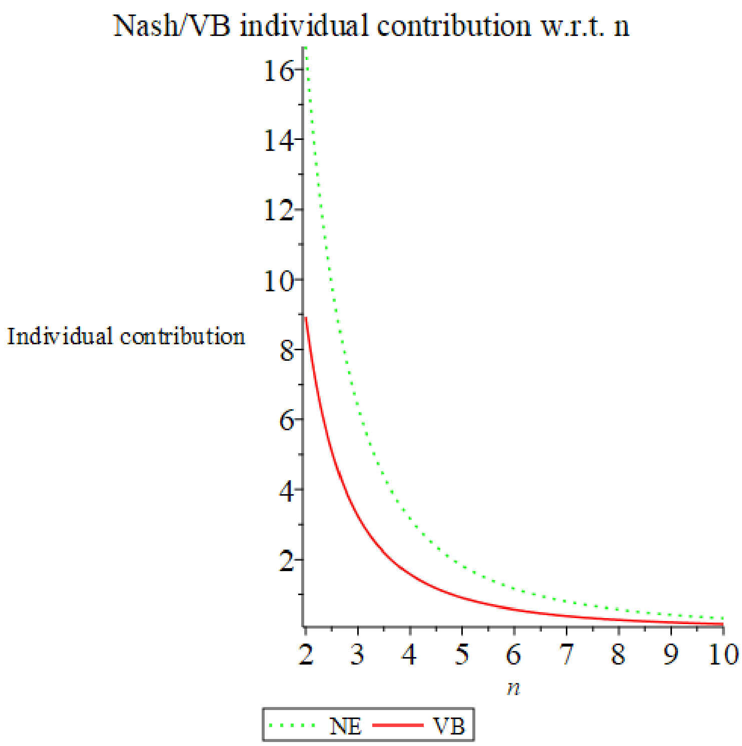

Figure 1.5: Nash and Virtual Bargaining contributions as number of players varies.

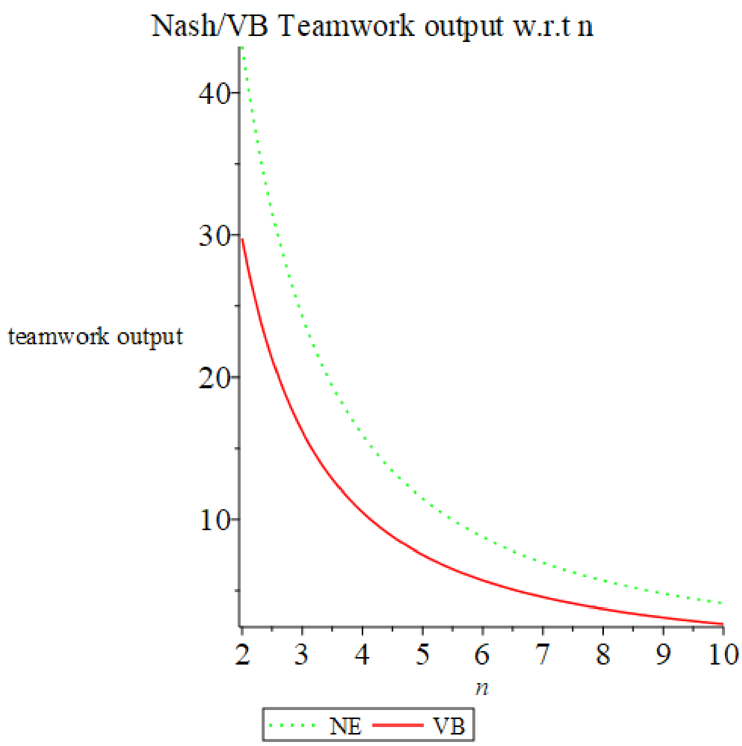

Figure 1.6: Total Nash and Virtual Bargaining contributions as number of players varies.

Chapter 2

An Experiment Test of Virtual Bargaining in Team Production Game and Price-setting Duopoly Market

2.1 Introduction

In this chapter, we discuss experiments to gain insight on the predictive power of the virtual bargaining theory in two contexts where the actions of agents are strategic complements. The first is a team production setting, which allows a test of the theory developed in Chapter 1. The second is a test of Melkonyan et al. (2017), which considers a duopoly market characterized by Bertrand competition. These experiments will be conducted when the Covid-19 pandemic no longer prevents experimental laboratory operations.

We focus on testing three aspects of the VB theory. First, we test whether people's contribution levels are significantly higher than the Nash prediction in the teamwork setting, as the Virtual Bargaining theory predicts. Second, we test key comparative statics developed from the theoretical model. For example, as the number of players increases in a group while other factors are constant, will players' contributions decrease as the theory predicts? Also, as the degree of strategic complementarity increases, will players' contributions increase as the VB theory predicts? Motivated by Melkonyan et al. (2017), we also plan to run a 20-period one-shot price-setting duopoly game. In this setting, the Virtual Bargaining model explains why people often see tacit collusion behavior in the Bertrand model (where firms are competing with prices) but not the Cournot model (where firms are competing with quantities). The key impacter that is driving the tacit collusion behavior is the strategic complementarity of prices in the Bertrand market. Firms are competing with production quantities in the Cournot model, which is strategic substitutes. The experimental test of collusion in a price-setting duopoly market serves as a comparison with the teamwork production game. Given similar theoretical predictions, we test whether similar decisions in experiments of different environments (teamwork production game vs. Bertrand game)?

In addition to testing the predictions of the Virtual Bargaining theory, our experiment is related to the literature studying how the strategic environment impacts individual behavior and, consequently, aggregate outcomes in various economic settings. Haltiwanger and Waldman (1985, 1989, 1991) theoretically showed how aggregated equilibrium with heterogeneous agents regarding information processing ability is dependent on the nature of the strategic environment. They showed that when the environment exhibits strategic substitution (termed a congestion effect in their model), then the agents with unlimited information processing ability (which they call sophisticated agents who can form rational expectations) tend to have a disproportionately large effect. When the environment exhibits strategic complementarity (termed a synergistic effect in their model), then it is the agents with limited information processing ability (which they call naive agents who form incorrect expectations) who have a disproportionately large effect. Motivated by Haltiwanger and Waldman (1989), Fehr and Tyran (2008) experimentally investigated the impact of bounded rationality under different strategic environments in the problem of the adjustment of nominal prices after an anticipated monetary shock. They have two versions of price-setting games, where participants' choices of price exhibit strategic substitutes in one treatment, and exhibit strategic complements in the other treatment. The strategic environment is the only difference between the two treatments. The game was repeated for a finite time, and the experiment was divided by a nominal shock into a pre- and post-shock phase of equal length. They find that the adjustment is significantly faster when price choices are strategic substitutes compared to strategic complements. However, in their set-up, the equilibrium is Pareto-efficient by construction as they want to rule out collusion and conditional cooperation as a decelerator in adjustment toward equilibrium. So, the collusion/cooperative effect is completely ruled out, while the incentive for collusion/cooperation is the central focus of the Virtual Bargaining theory. Another main difference is that theory of heterogeneity in information processing ability relies on a small portion of people who are boundedly rational (or have limited ability for information processing) in order for the theory to work in the environment of strategic complementarity. Exactly how big the portion is and how limited they are in terms of processing information has to be determined exogenously and will impact the prediction of the model.

Another related literature discusses about cognitive hierarchy and level-k reasoning Nagel (1995); Stahl II and Wilson (1994); Camerer et al. (2004). Camerer and Fehr (2006) summarized experimental data from various "p-beauty contest" game, and they find participants choose numbers that are significantly away from the Nash prediction. In one typical "p-beauty context" experiment Camerer and Fehr (2006), participants are asked to choose an integer number between 0 and 100. The participant whose number is closest to 0.7 of the average wins a fixed prize. The most common choice was 35, which is a good choice if you believe that the choices of others are random. In the beauty contest game, participants' choice of number exhibits strategic complementarity, such that if you believe other participants choose numbers higher than Nash equilibrium (zero), you have the incentive to follow by choosing a positive number. The results from the "p-beauty contest" support their cognitive hierarchy theory (Camerer et al., 2004, 2003). They argue that people have limited and various levels of iterated reasoning, and people believe that other people have less sophisticated reasoning levels than themselves. For example, if one person has 1 step of the iteration, then he may believe that other people would have 0 steps of iteration. In the "p-beauty contest" game, 0 step people would randomly choose a number between 0 and 100, then based on the belief that 0 step people randomize, the 1 step person should choose $0.7 \times 50 = 35$. In such a model, even for those people who are highly rational (meaning a high level of iterated reasoning), they choose numbers away from the Nash prediction as long as they believe there are other people who have less sophisticated iterated reasoning level.

The cognitive hierarchy theory is related to the limited ability in information processing theory. While the limited information processing theory assumes that there are two types of people, rational and boundedly rational, the cognitive hierarchy framework is more general in the sense that it assumes a more continuous distribution about the iterated reasoning level of people. Instead of two types of people, cognitive hierarchy assumes k types of people of various levels of iterated reasoning. Nevertheless, the cognitive hierarchy theory depends on the specification of the parameter k (highest level of reasoning) and the distribution of reasoning level among the population and, of course, the starting point (the assumption made about level-0).

Potters and Suetens (2009) experimentally studied whether strategic substitutability/complementarity has an impact on the tendency to cooperate in finitely repeated two-player games, where the Nash equilibrium is Pareto-inefficient. They have four treatments. Two are strategic substitutes with positive/negative externalities; The other two are strategic complements with positive/negative externalities. Because they are only interested in the effect of the strategic environment, in order to control for the framing effect, they use neutral labeling instead of price or quantity labels. They find that there is significantly more cooperation with strategic complements. However, although Potters and Suetens (2009) did find evidence to support the relationship between incentive for cooperation and strategic complementarity, they did not investigate the underlying mechanism of coordination. Chen and Gazzale (2004) studied the impact of the degree of strategic complementarity with regard to convergence to efficient Nash equilibria. They find that a higher degree of strategic complementarity implies significantly better convergence. However, as they are interested in the speed of convergence, the Nash equilibria are Pareto-efficient by construction. The potential for coordination/collusion are excluded by the experiment as in Fehr and Tyran (2008). Lappalainen (2018b) also experimentally investigate the impact of the strategic environment in voluntary contribution games. Besides the more commonly used standard linear setting (participants' choices are strategic substitutes), they implement the non-linearity in the production function of the joint project (where the participants' choices are strategic complements). They find that the rate of over-contribution compared to Nash prediction is much higher under the complementarity technology. However, as in Potters and Suetens (2009), Lappalainen (2018b) did not explain the underlying mechanism of over-contribution in complementarity technology. One of the main goals of our experiment is to investigate whether Virtual Bargaining theory as a mechanism has good predictive power in the setting of strategic complements.

Since, in our experiment, we also test the Virtual Bargaining theory in price-setting games, another literature related to our experiment are experiments studying collusion in price-setting games such as Potters and Suetens (2013). Suetens and Potters (2005) surveyed oligopoly experiments and found that there is significantly more tacit collusion in Bertrand than in Cournot markets (also see Holt (1995)). Later, Anderson et al. (2010) experimentally investigated the effect of strategic complements/substitutes in price-setting duopoly markets (Bertrand markets). This work specifically considered Bertrand substitutes (when products are substitutes, and price choice is strategic complements) and Bertrand complements (when products are complements, and price choice is strategic substitutes). The results contradict previous findings in that they find a moderate level of tacit collusion in Bertrand complements (when the price choices are strategic substitutes) but no evidence of collusion in Bertrand substitutes (when the price choices are strategic complements). Motivated by Melkonyan et al. (2017), whose theoretical results suggest a higher level of tacit collusion in the environment of strategic complements (price-setting game instead of quantity setting game), one part of our experiment is aimed at testing whether Virtual Bargaining theory can explain the tacit collusion in Bertrand market.

2.2 Experimental Design

The model developed in Chapter 1, together with Melkonyan et al. (2017) generate several testable hypotheses. We start by discussing the basic decision environment in the teamwork environment. Then we will discuss the decision setting of the Bertrand model. Following that, we will discuss the treatments and hypotheses being tested. Detailed instructions are attached at the end of the paper in Appendix B.1.

2.2.1 Decision Situation—Virtual Bargaining in Teamwork Game

In this section, we detail the theoretical predictions of traditional Nash theory and Virtual Bargaining theory [1] in the context of teamwork game. We start by discussing the general teamwork game structure. In the next subsection, we will discuss how the game develops under the traditional Nash theory. After that, we will discuss how the game develops under VB theory. The teamwork game works as follows. In a one-shot symmetric simultaneous game, N identical players are endowed with y_i endowment. Players simultaneously choose how much of their endowment to be contributed to the teamwork, σ_i, and they keep what is left, $y_i - \sigma_i$. The production technology of the team project exhibits strategic complementarity. That is saying that as some group members increase their contributions, other group members have the incentive to move in the same direction, increasing their contributions. All players equally share the return from the team project. The payoff function for each participant can be stated as

$$u_i(u_i, u_{-i}) = y_i - \sigma_i + M\sigma_i^{\frac{\alpha}{n}}(\prod_{j\neq i}\sigma_j^{\frac{\alpha}{n}}) \tag{2.1}$$

where the term $y_i - \sigma_i$ denotes the endowment that the player keeps for himself, and the term $M\sigma_i^{\frac{\alpha}{n}}(\prod_{j\neq i}\sigma_j^{\frac{\alpha}{n}})$ denotes the return from the team project, which is a function of all group members' contributions.

One key characteristic of such a game under the traditional Nash theory is that players have incentives to "free ride" on other players in the same group as the teamwork can be seen as a public good. However, based on different theoretical predictions, the level of "free-riding" is different. Under the traditional non-cooperative Nash theory, when players best responding based on other players' each possible strategy, the degree of "free-riding" is high, or we can say players are non-cooperative. Under the Virtual Bargaining theory, the predicted contribution level is higher than the Nash prediction (players are more cooperative). The intuition is that instead of best responding to other players' strategies, under the Virtual Bargaining theory, players would consider the possibilities of coordination and the associated payoffs depending on whether other players follow through by playing cooperatively (contributing higher than

[1] We will refer to VB instead of Virtual Bargaining in the following context.

Nash prediction) or taking advantage of your cooperative strategy (best responding to your strategy). With strategically complementary production technology, one player's cooperative strategy generates incentives for other players to mimic the strategy. Thus, a VB equilibrium where players contribute more than Nash prediction (coordinate) is achieved. In the following two subsections, we will further detail the traditional non-cooperative Nash theory and the cooperative VB theory in the context of teamwork game.

Traditional Nash Theory in Teamwork Game

Given the foundations, we develop the players' contribution strategies under the traditional Nash theory model. With each player's payoff function stated as (2.1), then maximizing the payoff for each possible strategy of other players, we obtain player i's best response function.[2]

$$R_i(\sigma_{-i}) = (\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} (\prod_{j\neq i} \sigma_j)^{-\frac{\alpha}{\alpha-n}} \tag{2.2}$$

In an N-player simultaneous-move game, when the initial endowment is large enough to ensure an interior solution when all players are playing their best response strategy, we obtain the Nash equilibrium as

$$\sigma^{NE} = (\frac{\alpha}{n})^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}} \tag{2.3}$$

Virtual Bargaining Theory in Teamwork Game

Under the VB theory model, players would consider each possible coordination strategy profile. The VB theory names the each possible coordination strategy profile as a candidate agreement. For each candidate agreement, players consider the two scenarios. One is if all other players follow through by playing the strategy suggested by the candidate agreement. The other one is if all other players best respond to the candidate agreement (taking advantage of the player who follows through the candidate). Then, players use the worst payoff of the two scenarios to guide their behavior. In the VB model, we construct player i's worst-payoff function as

[2]With the symmetric game, we can constraint our consideration to all other players playing the same strategy without loss of generosity.

$$w_i(\sigma_i^A, \sigma_j^A) = \begin{cases} y_i - {\sigma_i}^A + M{\sigma_i}^{A\frac{\alpha}{n}}(\prod_{j\neq i}{\sigma_j}^{A\frac{\alpha}{n}}) \\ \qquad\qquad\qquad \forall \sigma_i^A > M^{-\frac{n}{\alpha}}\alpha^{-\frac{n}{\alpha}}(\prod_{j\neq i}{\sigma_j}^A)^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}} \\ y_i - {\sigma_i}^A + \frac{\alpha}{n}^{-\frac{\alpha(n-1)}{\alpha-n}} M^{-\frac{\alpha(n-2)+n}{\alpha-n}} {\sigma_i}^{A^{-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2}}}(\prod_{j\neq i}{\sigma_j}^A)^{-\frac{\alpha^2(n-2)}{\alpha n-n^2}} \\ \qquad\qquad\qquad \forall {\sigma_i}^A \leq M^{-\frac{n}{\alpha}}\alpha^{-\frac{n}{\alpha}}(\prod_{j\neq i}{\sigma_j}^A)^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}} \end{cases} \tag{2.4}$$

where $j \in -i$. The superscript A denotes the strategies in candidate agreements.

The first piece in the equation is the case when all other players follow through the candidate agreement generates the worst payoff, so player i uses the associated worst payoff to guide his behavior. The second piece of the piece-wise equation is the case when all other players best respond to the candidate agreement generates the worst payoff, so player i uses the associated worst payoff to guide his behavior. [3]

Then, given other players' strategies in candidate agreements, finding the player i's strategy that maximizes player i's worst payoff, we obtain the player i's VB best response function.

$$R_i^F(\Delta) = \begin{cases} (\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} \Delta^{-\frac{\alpha}{\alpha-n}} \\ \qquad\qquad \forall \quad \Delta \leq \frac{\alpha}{n}^{-\frac{n-1}{\alpha-1}} M^{-\frac{n-1}{\alpha-1}} \\ (\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}} \\ \qquad\qquad \forall \quad \frac{\alpha}{n}^{-\frac{n-1}{\alpha-1}} M^{-\frac{n-1}{\alpha-1}} < \Delta \\ \qquad\qquad\qquad \leq (-\frac{\alpha n-n^2}{\alpha^2(n-2)+\alpha n})^{\frac{\alpha(n-1)}{\alpha n-n}} \frac{\alpha}{n}^{\frac{(\alpha-n)(n-1)}{\alpha n-n}} M^{-\frac{n-1}{\alpha-1}} \\ (-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2})^{\frac{\alpha n-n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \frac{\alpha}{n}^{-\frac{\alpha n(n-1)}{\alpha^2(n-2)+2\alpha n-n^2}} M^{-\frac{\alpha n(n-2)+n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \Delta^{-\frac{\alpha^2(n-2)}{\alpha^2(n-2)+2\alpha n-n^2}} \\ \qquad\qquad \forall \quad \Delta > (-\frac{\alpha n-n^2}{\alpha^2(n-2)+\alpha n})^{\frac{\alpha(n-1)}{\alpha n-n}} \frac{\alpha}{n}^{\frac{(\alpha-n)(n-1)}{\alpha n-n}} M^{-\frac{n-1}{\alpha-1}} \end{cases} \tag{2.5}$$

where $\Delta = \prod_{j\neq i}\sigma_j^A$ denoting the product of all other players' contribution in candidate agreement.

[3] Note that as players consider all possible candidate agreements, including symmetric and asymmetric ones. Thus, when in the candidate agreements such that the suggested contributions by other players are low and the suggested contribution by player i is high, then the case where all other players following through the candidate agreement could generate even lower payoff than if they all best responding.

In an N-player simultaneous-move game, when the initial endowment is large enough to ensure an interior solution, and when all players are playing their VB best response strategy, we obtain the VB equilibrium as [4]

$$\sigma^{VB} = (-\frac{\alpha^2(n-2)+\alpha n}{\alpha n - n^2})^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}} (\frac{\alpha}{n})^{-\frac{\alpha(n-1)}{(\alpha-1)(\alpha(n-2)+n)}} M^{-\frac{1}{\alpha-1}} \tag{2.6}$$

It is proved that the VB equilibrium contribution is higher than the Nash equilibrium contribution, $\sigma^{VB} > \sigma^{NE}$ (see Appendix A.4).

Decision Situation for The Teamwork Game Experiment

In this subsection, we discuss the basic decision situation in the teamwork game experiment. The participants are randomly assigned to a group of either two or three (differing by treatment). Each participant is endowed with 12 tokens, which he or she can either contribute to the team project or keep. The payoff function for each participant is stated as

$$u_i(\sigma_i, \sigma_j) = 12 - \sigma_i + M\sigma_i^{\frac{\alpha}{n}} \left(\prod_j \sigma_j^{\frac{\alpha}{n}}\right) \tag{2.7}$$

Where σ_i denotes the participant i's contribution to the team project. Thus, $12 - \sigma_i$ denotes the number of tokens the participant chooses to keep. M represents the scalar for the teamwork return. α represents the degree of complementarity of players' contributions. Thus, when $0 < \alpha < 1$, the return from the teamwork exhibits decreasing return to scale. We limit our discussion to this case, consistent with the focus of the theory. N is the number of participants in one group.

In sum, as we aim to test the VB theory against the traditional Nash theory, the design of the teamwork game experiment satisfies the following requirements. (1) In the control and treatment groups, the direct changes of Nash and VB predicted equilibria are the opposite. (2) In the treatment aimed to test the comparative statics of varying the number of group members, the predicted Nash and VB equilibria are both lowered by at least two tokens compared to the baseline treatment. (3) In the treatment aimed to test the comparative

[4]There are two feasible agreements under the VB model. The lower one coincides with the Nash equilibrium. The higher one is the VB equilibrium. For detailed discussion, please refer to section 1.5.

statics of varying the degree of complementarity, α, the predicted Nash and VB equilibria are both changed by at least two tokens.

2.2.2 Decision Situation—Virtual Bargaining in Bertrand Market

In this section, we detail the theoretical predictions of the non-cooperative Nash and VB theories in the context of price-setting duopoly market game (Bertrand game). We first discuss the general game structure. In the next subsection, we will discuss the equilibrium under the Nash theory. After that, we will discuss the equilibrium under the VB theory. The game works as follows. In a market of two firms, $i = 1, 2$, selling partially differentiated products, each of them is facing a linear demand, q_1 and q_2 respectively. The two firms must simultaneously and independently decide their respective prices for their products, p_1 and p_2. The direct demand functions for the two firms are stated as

$$q_1 = a - bp_1 + cp_2 \tag{2.8}$$

and

$$q_2 = a - bp_2 + cp_1 \tag{2.9}$$

where $b \geq c > 0$. Thus, the products the two firms are selling are substitutes.

As the marginal costs are normalized to zero in the model, the firm's objective functions are their the revenue functions.

$$\pi_i = ap_i - bp_i^2 + cp_ip_{-i} \quad i = 1, 2 \tag{2.10}$$

Both firms choose their price in order to maximize their revenue. However, under different theoretical models, the predicted price decisions are different. Similarly to the teamwork game in the previous section, the Nash predicted prices are lower than the VB predicted prices. Similar logic applies as well. In the Bertrand duopoly market game with substitute products, when one firm increases its price, the marginal revenue for the other firm is increased. The firm's cross partial derivative can be stated as

$$\frac{\partial^2 \pi_i}{\partial p_i \partial p_{-i}} = c > 0 \quad (2.11)$$

, and it is obviously positive. Thus, the other firm has the incentive to move in the same direction as one firm increases its price. With this property, under the VB model, the firms know that when they choose cooperative price levels (higher than Nash prediction price), the resulting payoff even if the other firm best responds would not generate a substantial revenue loss for the firm itself. When both players reason in such a way, a cooperative equilibrium can be achieved, which is the VB equilibrium.

Traditional Nash Theory in Bertrand Duopoly Market

Under the traditional Nash theory model, firms choose the price that maximizes their total revenue, given the other firm's price decision. We use the best response function to describe the relationship between the firm's choice of price, given the other firm's price decision. Then firm i's best response function can be stated as

$$R_i(p_{-i}) = \frac{a}{2b} + \frac{c}{2b} p_{-i} \quad (2.12)$$

When both firms are playing best response strategy, we obtain the Nash equilibrium price.

$$p^{NE} = \frac{a}{2b - c} \quad (2.13)$$

Virtual Bargaining Theory in Bertrand Duopoly Market

Under the Virtual Bargaining theory model, the logic is similar as in the teamwork game. Firms consider each possible coordination strategy profile, which is termed candidate agreement by the VB theory. For each candidate agreement, firms consider two scenarios. One scenario is when the other firm follows through the candidate agreement; The other scenario is when the other firm best responds to the candidate agreement. Firms would use the worst payoff among the two scenarios to guide their behavior. Based on the VB foundation, firm i's objective function/worst payoff function can be stated as

$$w_i(p_i^A, p_{-i}^A) = \begin{cases} (a - bp_i^A + cp_{-i}^A)p_i^A & \text{if } p_i^A > \frac{2bp_{-i}^A - a}{c}; \\ (a - bp_i^A + \frac{c(a+cp_i^A)}{2b})p_i^A & \text{if } p_i^A \leq \frac{2bp_{-i}^A}{c} \end{cases} \tag{2.14}$$

The first piece in the piece-wise equation represents the scenario when the other firm follows through the candidate agreement that generates the worst payoff. The second piece in the piece-wise equation represents the scenario when the other firm best responds the candidate agreement generates the worst payoff.

Given the other firm's strategy, p_{-i}^A, finding the price that maximizes firm i's worst payoff, we obtain firm i's VB best response function.

$$R_i(p_{-i}) = \begin{cases} \frac{a+cp_{-i}}{2b} & \text{if } p_{-i} \leq \frac{a}{2b-c} \\ \frac{2bp_{-i}-a}{c} & \text{if } \frac{a}{2b-c} < p_{-i} \\ \frac{a(2b+c)}{2(2b^2-c^2)} & \text{if } p_{-i} \geq \frac{a(4b^2-c^2+2bc)}{4b(2b^2-c^2)} \end{cases} \tag{2.15}$$

When both the firms are playing VB best response strategy, and their belief about the other firm coincides with the other firm's strategy, we get the VB equilibrium.[5]

$$p^{VB} = \frac{a(2b+c)}{2(2b^2-c^2)} \tag{2.16}$$

Comparison of equations (2.13) and (2.16) shows that the VB equilibrium price is higher than the Nash equilibrium price, $\sigma^{VB} > \sigma^{NE}$ (see Appendix B.2).

Decision Situation for Price Setting Duopoly Market Game Experiment

In this subsection, we discuss the basic decision situation in a symmetric duopoly price-setting game with partially differentiated products. Each player simultaneously chooses the price at which they sell their output, knowing how their own price and that of the other player will result in sales according to the demand system. To be consistent with the basic

[5]Like in the teamwork game, there are two feasible agreements under VB model in the Bertrand game as well. The lower one coincides with the Nash equilibrium. The higher one is the VB equilibrium. (Melkonyan et al., 2017)

setup in this experiment, the choice space is between \$0 and \$12 with integer allowed only. The demand system which determines the sales is as follows.

$$q_i = a - bp_i + cp_{-i} \tag{2.17}$$

where q_i denotes the quantity demanded for the associated firm, i, p_i denotes the price the associated firm chooses. With the aim to test the VB theory against the Nash theory in the context of Bertrand duopoly market game, the design of the experiment satisfies the following requirement: Compared with the control group, the Nash predicted equilibrium price is lower in the treatment group, while the VB predicted equilibrium price is higher in the treatment group.

2.2.3 Parameters and Testable hypotheses

In Table 2.1, we summarize the detailed treatment parameters and the associated Nash and VB theory predictions. The first treatment works as the baseline treatment. In the second treatment, we increase the scalar M and decrease the complementarity level α, such that the predicted Nash equilibrium is higher than baseline treatment, while the predicted Virtual Bargaining Equilibrium level is lower than the baseline treatment. The logic is as follows. The parameter α measures the degree of complementarity of contributions so that higher α generates a higher incentive for cooperation under the VB model. Thus, higher α means a higher relative magnitude of the VB predicted equilibrium contribution compared with the Nash predicted equilibrium contribution. Parameter M is the scalar for the return from the teamwork. Thus, a larger M means higher Nash and VB predicted equilibrium contributions while the relative magnitude is holding constant. In baseline treatment, α is relatively big (0.6), and M is relatively small (6.5). Thus the difference between the Nash and VB predicted equilibria is relatively big (4 tokens). Nevertheless, in the second treatment, α is relatively small (0.1), and M is relatively large (125), such that the Nash and VB predicted equilibria are almost identical. Comparing the second treatment with the baseline treatment, we directly test the Virtual Bargaining prediction relative to the Nash prediction.

In the third and fourth treatments, we test two of the comparative statics. In the third treatment, we increase the number of players (from two players in the baseline treatment to three). By comparing the third treatment with the first treatment, we test whether players' contributions decrease as more people are in the same group. Intuitively, more people in the same group means a greater difficulty of cooperation or a higher level of "free-riding" problem. In the fourth treatment, we decrease the complementarity level (from $\alpha = 0.6$ to 0.53). As the theory states, the predicted Nash and Virtual Bargaining equilibria will decrease as the degree of complementarity of contributions decreases. This is because that holding other factors constant, lower α means not only a lower degree of strategic complementarity but also a lower return from the teamwork project. So naturally, a lower α generates lower incentives for players to contribute in both the theory models (Nash and Virtual Bargaining).

Treatment five serves as the baseline treatment for the price-setting duopoly market game, where both the predicted Nash and Virtual Bargaining equilibria are 9 tokens. In treatment six, the predicted Nash equilibrium decreases to 7 tokens, but the predicted Virtual Bargaining equilibrium increases to 11 tokens. The critical factor that impacting the difference between the Nash and Virtual Bargaining equilibria is the relative size of the parameters b and c. Parameter b represents how fast the demand decreases as the firm increases its own price, while parameter c represents how fast the demand increases as the other firm increases its price level. Thus, under the constraint of Bertrand model that $b \geq c$, the closer b and c, the less differentiated the products are between the two firms (the higher degree of complementarity between the price decisions of the two firms), then the greater incentives for firms to collude under Virtual Bargaining theory. The larger b compared to c, the more differentiated the products between the two firms (the lower degree of complementarity between the price decisions of the two firms), then less incentive for firms to collude under Virtual Bargaining theory. Then in treatment 5, b is much larger than c, then both the predicted Nash and Virtual Bargaining equilibria are equal to 9 tokens. In treatment 6, $b = c = 2$, the products are perfect substitutes, then the predicted Virtual Bargaining equilibrium (11 tokens) is much higher than the Nash predicted equilibrium (7 tokens). What's more, the parameter a impacts the overall market scale and, thus, the absolute level of Nash (Virtual Bargaining) equilibria. Comparing the treatment 5 and 6, we can test whether the Nash or Virtual Bargaining theory has better predictive power in the context of price-setting duopoly market game.

Below we summarize the key hypotheses to be tested by the experiment:

Hypothesis 1: Players' contributions to the team project will be strictly higher than the Nash equilibrium prediction as predicted by the Virtual Bargaining theory.

Hypothesis 1' : Players' choices of prices in the Bertrand game will be strictly higher than the Nash equilibrium prediction as predicted by Virtual Bargaining theory.

Hypothesis 2: Players' contributions in treatment 2 will be strictly lower than the contributions in treatment 1, consistent with the Virtual Bargaining theory and contrary to Nash equilibrium predictions.

Hypothesis 3: Players' contributions will be lower when the number of players is three (in treatment 3) than when the number of players is two (in treatment 1).

Hypothesis 4: As the complementarity degree decreases, we expect the players' contributions to decrease. (in treatment 1 and 4)

Hypothesis 5: We expect the subjects' price decisions in the less differentiated-product price-setting duopoly market (treatment 6) to be significantly higher than in the more differentiated-product market (treatment 5) as predicted by the Virtual Bargaining theory.

2.2.4 Experimental Procedures

The experiment will be conducted with the software z-tree Fischbacher (2007, see). The experimental instructions are summarized in Appendix B.1. The number of participants in each treatment in the teamwork game will be determined when we complete the pilot session as there are no closely related teamwork game experiments. The number of participants in each treatment in the price-setting duopoly market game is set to be 28 per treatment for the pilot session based on the data from a related experiment Potters and Suetens (2009). For a detailed discussion, please refer to section 2.3. But the required participants number per treatment may change after we collect data from the pilot session.

The participants in all treatments in the teamwork game experiment receive the same instructions (see Appendix B.1). The instructions for treatments in teamwork game experiments only differ with respect to the payoff function parameters. The participants in all treatments in the Bertrand duopoly market game receive the same instructions, with differences only with respect to the demand equation parameters. It will be explained to the participants that their payoff is dependent on their own choices and also one (two) other participant(s) in the same group. Participants will be randomly re-matched for each decision period with the aim of reducing the reputation effect/learning effect. The game will run for 20 periods for each treatment, which we expect to be enough for participants' behavior to converge to an equilibrium. There is one practice period before the paid period, which doesn't count to calculate earnings. After each decision period, participants are informed about their own and other group members' choices, and also their payoff in experimental tokens. Before the main experiments, we ask the participants to answer multiple practice questions in order to make sure that they understand the game. They are compensated with 10 tokens for each correctly answered practice question. Earnings are calculated in experimental tokens first and then transformed into US dollar at the exchange of \$1 = 25 experimental tokens for the teamwork game experiment (\$1 = 120 experimental tokens for the Bertrand duopoly market game experiment) at the end of the experiment. The expectation of the duration of the experiment will be based on a pilot session. We anticipate the average earnings for each participant to be \$23, with the highest possible earning be \$34, and the lowest possible earning be \$0. At the conclusion of the experiment, participants will complete a short questionnaire, which includes basic demographics questions and the 10-item Big-Five personality instrument of Gosling et al. (2003), and additionally, two extra questions about their willingness to cooperate/work independently.

2.3 Statistical Methods

To determine the necessary sample sizes to test our hypotheses outlined above effectively, we plan to run a pilot session for the teamwork production game to inform power calculation. As for the Bertrand market game (price-setting duopoly market), we refer to a previous experiment Potters and Suetens (2009). They conducted a laboratory experiment to examine whether the strategic environment impacts the tendency to cooperate in a finitely repeated two-player game, where one of their treatment is a typical Cournot game, and another treatment is a typical Bertrand game. In their experiment, the decision range is from 0 to 28 (with allowed decimal point to be one). The reported standard deviation is 4.38. Based on that, as the choice range in our experiment is 0 to 12 tokens, we adjust the standard deviation to be $4.38 \times 12/28 = 1.88$. The reported intraclass correlation is 0.59. The group size is designed to be two. So, given the cluster size be two, at 80% power using a 5% significance level, the smallest effective size to be 2, the required sample size for each treatment is 24. Thus, having 28 participants in each treatment should generate enough power for the price setting duopoly market game. However, the required number of participants per treatment may change after we collect data from the pilot session.

The outcome measures that we are most interested in are the contribution level in the teamwork game, which we will use to compare with the theoretical prediction. In the price-setting duopoly market game, we are most interested in the price decisions. Also, by converting absolute contribution level in the teamwork game and price decisions in the price-setting duopoly market game into relative level with respect to predicted Nash and VB equilibria, we can use the data to analyze which theory yields better prediction. Once we have the data, we will use a linear regression model with cluster-robust standard errors clustered at the individual level. The included explanatory variables are the set of treatment indicators. By comparing the coefficient of the dummy variables, we will apply the paired t-test to test the hypothesis.

Social efficiency is another outcome measure that is of interest to us as well. In the teamwork game, social efficiency can be measured as the ratio of group payoff over the max possible group payoff. In the price-setting duopoly market game, as we consider both the suppliers and consumers' surplus, then the social efficiency can be measured as the ratio of the sum of group profits plus consumer welfare over the max possible sum of group profits plus max possible consumer welfare. What's more, we will also extend the model by including extra control variables from the questionnaire. One of the purposes of this is as a robustness check. Second, there might be interesting results associated with personalities. In addition to common issues like gender effects, we are also interested in whether people's self-claims about their willingness to cooperate are consistent with their behavior.

Table 2.1: Experiment parameter table

Teamwork Game					
Treatment	Parameters			Nash Equilibrium	Virtual Bargaining Equilibrium
	M	α	N		
1	6.5	0.6	2	5 or 6	10
2	125	0.1	2	8	8
3	6.5	0.6	3	1 or 2	4
4	6.5	0.53	2	3	5
Bertrand Game in Duopoly Market					
Treatment	a	b	c	Nash Equilibrium	Virtual Bargaining Equilibrium
5	24.4	1.6	0.5	9	9
6	14.4	2	2	7	11

Chapter 3

Optimal Contracting for Online Display Advertising

3.1 Introduction

Internet display advertising refers to a market for graphical and video ads displayed on websites for a targeted audience. Internet display advertising is related to but distinct from textual ads on search pages, where advertisers bid for the ranking of results shown on the search page. Based on the information about users' internet browsing history captured through the technology known as "cookies", internet display advertising gives advertisers opportunities to target users based on known characteristics. Advertisers can target a specific audience segment by restricting several characteristics of the recipient, such as demographics, geography, and browsing history. (For example, a pre-owned car dealership located in Knoxville may wish to target males also located in Knoxville or cities near Knoxville, aged from 17 to 70, with income ranged above $20,000.) Publishers (the owners of websites) gain revenue by selling a large number of impressions (or website user visits/ user's eyeballs). Each visit by one user on the website is called one impression. There are big publishers like Yahoo! Sites, Fox International Media, and also large numbers of small publishers like website owners of business or bloggers. By targeting a specific consumer group, internet display advertisers achieve greater efficiency in the advertisement market.

There are two main methods for publishers to sell website impressions. One is the traditional method, where an advertiser signs a contract with a publisher, which is called the guaranteed contract (or traditional contract). The other method is selling impressions through the spot market, where impressions are sold by real-time bidding online auction through ad exchange (Balseiro et al., 2014). Internet advertising revenues in the United States totaled $57.9 billion for half-year of 2019. The revenue increased 16.9% for the half-year of 2019 over the half-year of 2018 Bureau (2019). Around 80% of the total revenue in the market is contributed by the ad exchange market and in an increasing trend [1]. In this paper, we first illustrate the consequences of the moral hazard problem arising from asymmetric information in the traditional contract. We demonstrate that moral hazard is costly for the advertiser in this context. We then design an incentive-compatible contract for the publisher and the traditional contractor which can resolve the moral hazard problem that exists in this setting and induce the efficient level of effort by the publisher. Details of the nature of the moral hazard problem and the mechanism of the contract will be discussed later. We will discuss some important characteristics of the two markets in the following subsection to provide an understanding of the nature of the online display advertising markets.

3.1.1 The dual-market nature of the online display advertising market and the moral hazard problem

The traditional method (the guaranteed contract or traditional contract) usually takes the form of a specified number of impressions reserved over a particular time horizon, targeted to a specific segment of the audience by specifying consumer characteristics to receive the impressions such as gender, age range, location, browsing history and so on specified in the contract. As the name 'guaranteed contract' suggests, the publisher commits to deliver the exact number of impressions satisfying the requirements specified ex-ante in the contract. Typically, there will be specific penalties if the publisher fails to fully implement the contract (Yang et al., 2010). For example, a movie producer may want to purchase 10 million impressions two months before a movie be released, targeting the audience who visited film

[1] https://www.statista.com/outlook/216/109/digital-advertising/united-states#market-revenueYearIndustry

or entertainment websites relatively frequently. Typically, the probability that a user clicks on an ad (known as click-through rate) is used as a metric of placement quality (Balseiro et al., 2014). The guaranteed contract is a traditional method of selling advertising. The main advantage of the traditional contract is that they allow the advertiser to hedge against uncertainties in the supply of impressions in terms of both quantity and quality by signing a guaranteed contract beforehand.

The alternative to a guaranteed contract for a potential advertiser is to purchase impressions on the spot market, where impressions are sold through real-time bidding in an online auction. As the volume and complexity of the digital advertising industry grew, publishers had difficulty maintaining efficiency when dealing with large quantities of advertisers. There emerged an intermediary called Ad Network, which aggregates available impressions across different publishers and sells them to different advertisers. Ad Network collects impressions and allocates them to the advertising market while satisfying the various requirements of different advertisers. Online publishers rely on advertising networks to sell inventory that has not been sold directly and also as a substitute for direct selling in some cases (Evans, 2009). Later a new type of technology-driven intermediary known as ad exchange (or AdX) arrived as the real-time bidding technology develops. The Ad Exchange works like a stock exchange. It provides a platform for publishers and advertisers or even Ad networks to sell and purchase impressions in a real-time bidding auction form. Once a web user opens a website, the publisher posts this impression on the ad Exchange. Advertisers or Ad networks can bid for this impression opportunity. The bidder with the highest bid can display its ad to the web user and pay the price (depending on the exact form of auction, like the first- or second-price auction; both forms of auctions are used in the industry (Abraham et al., 2013)). This happens in a fraction of a second, just between the time the web user opens the website and sees the ad. Some famous examples are Google's DoubleClick, Microsoft's AppNexus (switched from AdECN in 2010), and Verizon Communications's AOL. The Ad exchange market gives advertisers the opportunity to target even more precisely on the audience. As the Ad Exchange sells impression by impression, the advertisers can decide whether to bid on every single impression available.

Both the two methods of selling impressions are available to publishers and thrive side by side (the guaranteed contract and Ad Exchange spot market). The spot market provides advertisers the opportunity to bid for very narrowly specified types of impressions, while the guaranteed contract provides advertisers the opportunity to hedge against risks in the supply of impressions. In the Ad Exchange, cookie information sharing is offered by most Ad Exchanges. By comparing the cookie information offered by Ad Exchange and the cookie information of advertiser's own, the advertisers have more precise information about impressions. For example, whether the user visited their website or its rivals' websites before or whether they have sent an ad to the user before, or what has the user searched on the advertiser's website. This information gives the advertiser the ability to target more precisely on the audience in the Ad Exchange market than just purchasing the guaranteed contract with the publisher. Then, when the number of buyers in the spot market is big enough, the auction price becomes an unbiased reflection of the true value of the impression. As for the publisher, when more than one impression satisfying the traditional contract's requirement is available to satisfy the contractual agreement, she must decide which impressions to be allocated to the traditional contract and which ones to be allocated to the spot market. When multiple impressions satisfying the requirements from the guaranteed contract are available, they might have different valuations to advertisers on the spot market as it is impossible for the ex-ante guaranteed contract to capture all the aspects of impressions impacting its value.

Given the spot market and the guaranteed contract's nature, it creates a circumstance involving moral hazard on the part of the publisher when allocating the impressions. As Balseiro et al. (2014) stated

> "In the presence of AdX, publishers face the multiobjective problem of maximizing the overall placement quality of the impressions assigned to the reservations together with the total revenue obtained from AdX, while complying with contractual obligations. These two objectives are potentially conflicting; in the short term, the publisher might boost the revenue stream from AdX at the expense of assigning lower-quality impressions to the advertisers. In the long

term, however, it may be convenient for the publisher to prioritize the advertisers in view of attracting future contracts."

As the compensation for satisfying the required consumer impressions in the guaranteed contract is determined ex-ante, the publisher has an incentive to allocate the relatively low-quality impressions (still satisfying the requirement of the traditional contract) to the traditional contractor in order to save high-quality ones for the spot market for a higher spot market expected revenue. What's more, the publisher has more information about the overall market situation, such as the quality of the overall supply side of impressions at the moment allocating impressions between the traditional contractor and the spot market. This information asymmetry limits the ability of traditional contractors to verify the publisher's effort in terms of providing high-quality impressions to them, and thus creates a challenge in design a contract to induce an efficient effort while limiting compensation to the publisher. The effort here could be understood as the quality of impressions allocated to the traditional contract relative to the average quality of impressions sold on the spot market. The impressions delivered to the traditional contractor with high click-through rates could be the result of a better overall quality of the supply of impressions, or it could be the result of the publisher's effort of allocating relative high-quality impressions to the traditional in a bad market state. The final verifiable quality of impressions is due to the combined effects of the publisher's real effort and the nature of the supply of impressions in the market. So this inability to match the payment with the publisher's real effort can lead to an inefficient level of effort exertion by the publisher and gives the publisher with good overall impressions supply the opportunity to gain positive rents for just being in a good time.

3.1.2 Literature Review

There are two papers explicitly focusing on the topic of moral hazard when a publisher is allocating impressions between a guaranteed contract and the spot market. The two pieces of literature address the moral hazard problem arising from information asymmetry by bringing either fairness or long-term benefits (like reputation among potential future contract buyers) into the publisher's objective function. Ghosh et al. (2009) in the International Workshop on Internet and Network Economics conference 2009 argued that the correlation between the price (the bid price in the spot market) and the value (the quality of the impressions) means the bidding price of an impression can reflect the unbiased value of the impression as the number of bidders is large enough. They argue that a perfectly representative allocation of impression to the guaranteed contract advertiser is one that at every price level, the same portion of impressions is allocated to the guaranteed contractor. Ghosh et al. (2009) proposed to use the randomized bidding strategy with a target budget to bid on behalf of the guaranteed contractors in order to deliver a set of impressions to the guaranteed contractors that are closest to the perfectly representative allocation. The publisher's choice of the target budget represents her trade-off between the spot market revenue and fairness. Balseiro et al. (2014) argued in Management Science that the conflict between the short-term revenue from ad exchange and the long-term benefits of delivering good spots to the reservation ads (the guaranteed contract) can be solved by modeling the publisher's multi-objective problem by taking a weighted sum of (i) the revenue from AdX and (ii) the placement quality of the contracts. They utilize the techniques of revenue management. They take the publisher's joint objective problem as a combination of capacity allocation problem to handle guaranteed contract and a dynamic pricing problem to handle the spot market. The publisher cares about the impression placement quality to the guaranteed contractor besides the short-run revenue, as that they care about their long-term relationship with the guaranteed contractor as well, like reputation. In general, these papers consider the publisher's objective when allocating impressions to maximize the spot market revenue and also considering the impression quality problem of the guaranteed contract, which is not monetary. However, their objective is what they think what they should do but not necessarily incentive-compatible if they only aim to maximize their monetary revenue. Then, given the objective, they discussed what kind of allocation mechanism the publisher can adopt to achieve the goal. We show that even when the publisher is only interested in maximizing her monetary revenue, we could design an incentive-compatible contract between the publisher and the guaranteed contractor such that the induced effort by the publisher is socially efficient (maximizing the combined welfare of the spot market and the guaranteed contract). Put it in another way; this contracting mechanism is designed from the perspective of the traditional contract advertiser such that even the monetary-maximizing publisher finds it is her best interest to exert the socially efficient level of effort. At the same time, the traditional contract advertiser can avoid paying rents for the information advantage the publisher has. The results doesn't depend on the publisher's self-discipline.

Another strand of literature that is related to ours but not as direct as what is discussed in the above paragraph are studies about auction/market design problems in the internet ads market. Yokoo et al. (2004) studies the effect of false-name bids in combinatorial auctions, which are used in the internet auction. Babaioff et al. (2010) treated the uncertain impression supply as identical items arriving dynamically and study mechanisms with the goal to maximize social welfare. Arnosti et al. (2016) introduced the "modified second bid" auction that overcomes the inefficiency and adverse selection problem of the second-price auction when advertisers' valuation for impressions are positively correlated, false-name bidding problem, and the complexity of Bayesian-optimal auction. Our paper contributes to the literature on the specific topic of solving conflicts between the spot market revenue and the traditional contract impression quality placement. In order to better characterize the moral hazard problem faced by the publisher when allocating impressions between the two markets, we first formalize the moral hazard problem when the traditional advertisers use the traditional contract that doesn't utilize any correlated information from the spot market. Based on that, we design an incentive-compatible contract that utilizes the price information from the correlated spot market, which addresses the moral hazard problem. What's more, we use simulation to graphically show the significant difference in terms of social efficiency and the traditional contractor's expected payoff based on the two contracting mechanisms.

Instead of bringing either fairness or future reputation into the model, we consider the problem of designing a socially efficient incentive-compatible contract for the advertiser and the publisher utilizing the pricing information from the spot market. The idea is that since the two markets share between them the entire supply of impressions created by the publisher, then there are correlations between the average quality of impressions delivered to the traditional contract market and the average bid price in the spot market. By making the payment to the publisher for the traditional contract dependent on the realized aggregated price in the spot market while the other portion of the payment is a function of the publisher's reported market situation, the contract can induce the publisher to exert the efficient level of effort while being compensated just enough for the effort (The effort here is associated with the opportunity cost of forgone revenues from allocating impressions to the spot market). The idea of using the information from the related market to extract full surplus was first studied in auction theory by Crémer and McLean (1988). They proved that when bidder's valuations are correlated, the seller can sell the object while extracting the full surplus as if the seller can sell the item with full information about the bidder. Later, this idea was implemented in the duopoly industry regulation problem. Bertoletti and Poletti (1997) showed that by introducing the availability of some signal on the rival's behavior, when costs are correlated, the regulator could induce the regulated firms to exert an efficient level of effort towards reducing production costs (see also Laffont and Tirole (1993)). In this paper, we use the power of information again to achieve a socially efficient outcome for the online display advertising market, while solving the moral hazard problem in the market.

3.2 The Model Setup And The Complete Information Case

We first consider a contract design problem faced by a traditional advertiser who contracts with the publisher for advertising impressions under asymmetric information about the state of the market. Therefore the effort exerted by the publisher is not directly observable. The traditional advertiser can not tell whether the realized impression quality is due to the good market state or the effort by the publisher. In addition to contracting with the traditional advertiser, the publisher also sells impressions through the real-time bidding market. The effort by publisher here measures the relative quality of impressions allocated to the guaranteed contract compared to the quality of impression sold through the real-time bidding market. We will discuss what the effort means in detail later when we introduce the model setup.

We assume that a traditional contract advertiser has contracted for a certain quantity of impressions, which are determined ex-ante, but the quality of those impressions is determined by the allocation of different impression opportunities available to the publisher. This allocation is not fully contractible. Let q, to denote the quality of impressions the traditional contractor receives from the publisher. The realized quality of impressions delivered to the traditional contractor is impacted by the overall market situation, which is a result of stochastic conditions, denoted by β. We assume that the overall market impression quality β (we will refer to it as the market state for the rest of this paper) follows a stochastic distribution on the support of $[\underline{\beta}, \bar{\beta}]$. $f(\beta)$ is the associated probability density function and $F(\beta)$ is the associated cumulative distribution function. Whatever the realized market state, the publisher can allocate impressions between the traditional contract advertiser and the spot market.

From the perspective of the publisher, when she is deciding which impressions to deliver to the traditional contractor, she must balance between impression quality allocated to the traditional contractor and allocated to the real-time bidding spot market. Even though specific requirements of impressions delivered to the traditional contractor are often determined ex-ante to meet the traditional contractors' specific needs (such as impressions being drawn from specific geographic locations, certain age range, or certain income range), there are often multiple impressions that satisfy those requirements. For this reason, when facing multiple impressions all satisfying the traditional contractor's requirement, the publisher has the incentive to allocate the ones with relatively higher valuations to the real-time spot market to get a higher expected revenue from the spot market while the revenue from the traditional contract is fixed as long as the impression requirements are met.

For a specific overall market state, we use the variable, e, to denote the relative quality of impressions allocated to the traditional contractor compared with the quality of impressions sold through the spot market. In this sense, e represents the effort the publisher exerts toward providing the traditional contractor with quality impressions. The higher the effort, e, the publisher exerts toward the traditional contractor, the higher the quality of the impressions delivered to the traditional contractor. Conditional on the market state, β, the higher the publisher's effort, e, the lower the quality of impressions allocated to the real-time bidding spot market. Let's consider the scenario when the impression traffic comes in, and the publisher is trying to allocate multiple impressions between the markets. The impressions have multiple dimensions that impact its valuation to advertisers, including but not limited to the gender, income range, location of the potential consumers viewing the advertising. Since the guaranteed contract is determined beforehand, the guaranteed contract can only capture a limited number of dimensions. Thus, when impression traffic comes in, there are multiple impressions satisfying the guaranteed contract's requirement but having different valuations and sold with different prices in the spot market due to the uncaptured dimensions of those impressions in the guaranteed contract.T publisher chooses to determine which will be delivered to the guaranteed contract and which ones sold through the spot market. Thus, effort by the publisher implies delivering the impressions with relative higher valuations to the guaranteed contract, and the associated cost of the effort is the forgone revenue from the spot market that could have been earned if the relatively higher valuation impressions were sold on the spot market.

We use function $q(\beta, e)$ to denote the absolute (not relative) quality level of the impressions delivered to the traditional contractor, which depends on the state of the market and the effort from the publisher. We assume that $q_1(\beta, e) = \frac{\partial q}{\partial \beta} > 0$ and $q_2(\beta, e) = \frac{\partial q}{\partial e} > 0$. These two assumptions imply the following. The better the overall market state, the higher the quality of impressions delivered to the traditional contractor for any given level of effort. Similarly, the more effort the publisher exerts to the traditional contractor, the higher the quality of impressions delivered to the traditional contractor for a given state of the market. The traditional contractor's monetary value of the impressions is normalized to equal the quality of the impressions, $q(\beta, e)$. The traditional contractor's net payoff is then the value of the impressions minus the monetary payment they make to the publisher.

The publisher's payoff is the sum of the monetary transfer she gets from the traditional contractor and the revenue from the real-time bidding spot market. We use $R(\beta, e)$ to denote the revenue for the publisher from the real-time bidding spot market. It is intuitive to have the two assumptions, $R_1 = \frac{\partial R}{\partial \beta} > 0$ and $R_2 = \frac{\partial R}{\partial e} < 0$. The two assumptions state that the better the overall state of the market, the higher the revenue from the spot market; and that the more effort the publisher exerts, the lower the revenue from the spot market.

For simplicity, we assume that $R_{12}(\beta, e) = 0$ and $q_{12}(\beta, e) = 0$. This assumption put some constraint on the functional form of $R(\beta, e)$ and $q(\beta, e)$ but still captures a large range of possibilities. In this paper, we specifically assume that $R(\beta, e) = s_2(\beta) - p_2(e)$ and $q(\beta, e) = s_1(\beta) + p_1(e)$, where the first-order derivatives of s_1, s_2. p_1 and p_2 are all positive.

Given $q(\beta, e) = s_1(\beta) + p_1(e)$, it is natural to assume that $q_{22}(\beta, e) = p_1^{''}(e) \leq 0$. This assumption implies that the quality of the impressions delivered to the traditional contractor, q, is weakly concave in the publisher's effort to the traditional contractor, e. In other words, the quality of impressions delivered to the traditional contractor is increasing in the effort, e, but is increasing at a non-increasing speed. Similarly, we also assume that $q_{11}(\beta, e) = s_1^{''}(\beta) \leq 0$. This assumption is saying that the realized impression quality is also weakly concave in the market state. The realized impression quality is increasing but at a non-increasing speed in the realized market state.

What's more, we assume $R_{22}(\beta, e) = -p_2^{''}(e) < 0$. This is saying that as the publisher exerts more and more effort, the lost revenue from the spot market is increasing at an increasing speed. In other words, the cost of putting effort into the traditional contractor is convex.

We now characterize the information structure and nature of the market interaction. The realization of the overall market state β follows a distribution, whose pdf and cdf are $f(\beta)$ and $F(\beta)$, respectively. After nature takes a draw from this distribution. The publisher has full knowledge of β, while the traditional contractor only knows the distribution from which β is drawn. Then the traditional contractor offers a contract to the publisher. The publisher can choose to accept or refuse. Once the publisher accepts, she observes the state of the market, β, reports a market state to the contract advertiser, and decides how much effort she exerts toward satisfying the traditional contract and consequently, the residual effort allocated to the spot market. Then, the traditional contractor observes the realized quality of impressions delivered to him, resulting from the combination of the state of the market and the publisher's effort, and finalizes the payment to the publisher according to the contract. The publisher observes the output and receives payment from the traditional contractor and also from the spot market.

We first consider the case of complete information when both parties can observe the realization of the market state, β, and thus, also the publisher's effort level, e. Then the traditional contractor can design the payment to the publisher such that the publisher's participation constraint is just binding, and she earns no information rents. Now we assume that the publisher's outside alternative is to allocate all the impressions to the spot market and refuse the traditional contract. In such a scenario, the publisher's total revenue is defined as $R(\beta, 0)$. In other words, given the market state β, exerting zero effort to the traditional contractor, the revenue the publisher gets from the spot market, $R(\beta, 0)$, is the outside option. So the disutility for the publisher from exerting effort to the traditional contract is the revenue loss from the spot market she could have earned if she had invested that effort into the spot market. It is defined as $R(\beta, 0) - R(\beta, e)$. With complete information, the traditional contractor can just pay the publisher the disutility of exerting e effort. Thus, under such a scenario the traditional contractor's objective function is

$$\max_{e} \; q(\beta, e) + R(\beta, e) - R(\beta, 0) \tag{3.1}$$

We accordingly define the optimal level of effort under complete information, $e_E^*(\beta)$ which maximizes the above objective function. At the level of $e_E^*(\beta)$, we have

$$q_2(\beta, e_E^*(\beta)) + R_2(\beta, e_E^*(\beta)) = 0. \tag{3.2}$$

As for the social planner, the objective is to maximize the sum of both parties' welfare. In our paper, the sum of both the parties' welfare is defined as the return from the impressions allocated to the traditional contract plus the revenue from the spot market. Thus, we have the social planner's objective function as

$$\max_{e} u_s = q(\beta, e) + R(\beta, e) \tag{3.3}$$

Maximizing the objective function w.r.t the effort level, we get the F.O.C

$$q_2(\beta, e^{**}) + R_2(\beta, e^{**}) = 0 \tag{3.4}$$

which is identical to the F.O.C of the complete information case when the traditional contractor can observe the realization of the market situation. So the efficient level of effort is also consistent with the social planner's objective to maximize social welfare. Please note that both e_E^* and e^{**} are independent from the market state, β, with the assumptions $R_{12}(\beta, e) = 0$ and $q_{12}(\beta, e) = 0$. We construct the game in such a way so that we can focus on the moral hazard problem. [2]

3.3 Isolated Market

In this section, we consider the contract design problem faced by the traditional advertiser under asymmetric information when the contract cannot be made contingent on the information from the spot market. In this case, the traditional contractor's payment to the publisher is contingent only on the market state reported by the publisher and the absolute quality of impressions the traditional contractor receives from the publisher. This model formalizes the notion that moral hazard in the presence of asymmetric information regarding the state of the market yields sub-optimal (second best) outcome for the traditional advertiser.

The traditional contractor only knows the distribution of the possible market states when offering a contract to the publisher. The publisher decides whether to accept the proposed contract or not. If she accepts, she will observe the market state before deciding what state to report to the contract advertiser and choosing her level of effort to the traditional contractor (i.e., how to allocate impression between the spot market and contract advertiser). The contract works as follows. First, the publisher announces the market state. We use $\hat{\beta}$ to denote the announced market state, which may or may not be true. Then, the contract specifies a required absolute impression quality that the publisher has to satisfy contingent on the reported state of the market. The required absolute impression quality is denoted by $\hat{q}(\hat{\beta})$. When the publisher delivers impressions satisfying the requirement $\hat{q}(\hat{\beta})$, a monetary transfer as a function of the announced overall market state will be made from the traditional contractor to the publisher. We use $t(\hat{\beta})$ to denote the associated monetary transfer for announced market state, $\hat{\beta}$. For identifying the publisher's optimal contract design, we apply the Revelation Principle, which states that, without loss of generality, an optimal

[2] For a detailed derivation of the result mentioned above, please refer to C.1.

contract can be found among the set that induces the publisher to truthfully reveal the state of the market. That is, we identify an optimal contract that is a mechanism to induce truthful revelation.

We now identify the publisher's payoff when the true market state is β, and when she announces $\hat{\beta}$. The publisher's utility as a function of the true parameter β and the announced $\hat{\beta}$ and the effort level e is

$$u_p(\beta, \hat{\beta}, e) \equiv t(\hat{\beta}) - (R(\beta, 0) - R(\beta, e)) \tag{3.5}$$

where $R(\beta, e)$ denotes the publisher's revenue from the spot market, and $R(\beta, 0)$ denotes the publisher's spot market revenue if she exerts no effort to the traditional contract. Naturally, $R(\beta, 0) - R(\beta, e)$ represents the publisher's loss of spot market expected revenue due to the effort she exerts to the traditional contract. Or we can see it as the opportunity cost of the effort, e. $t(\hat{\beta})$ is the monetary transfer the publisher gets from the traditional contractor.

The advertiser's payoff function is identified as follows. The traditional advertiser's payoff as a function of true parameter β, the announced parameter $\hat{\beta}$, and the effort by the publisher e, can be stated as

$$u_a \equiv q(\beta, e) - t(\hat{\beta}) \tag{3.6}$$

where the $q(\beta, e)$ denote the quality of impressions allocated to the traditional advertising contractor (traditional advertiser). We normalize the monetary payoff from the impressions to be the quality of the impressions.

3.3.1 The Publisher's Problem

When the publisher announces $\hat{\beta}$, then the required absolute impression quality provided to the traditional contractor is $\hat{q}(\hat{\beta})$. Therefore the publisher has to exert effort level e, such that

$$\hat{q}(\hat{\beta}) = q(\beta, e) \tag{3.7}$$

The above function implicitly defines the effort level that the publisher has to exert when she announces $\hat{\beta}$ while the true parameter is β. So we can invert the above function to explicitly define the effort level required as a function of the announced parameter $\hat{\beta}$, and the true parameter β. We use the following function to explicitly define the required effort level.

$$e = e(\hat{\beta}, \beta) \tag{3.8}$$

Replacing the effort level with the above effort function, then the publisher's payoff function can be stated as a function the true parameter β and announced parameter $\hat{\beta}$.

$$u_p(\beta, \hat{\beta}) \equiv t(\hat{\beta}) + R(\beta, e(\hat{\beta}, \beta)) - R(\beta, 0) \tag{3.9}$$

Truthful Revelation Requirement

A truthful revelation mechanism requires that the publisher finds it optimal to report the true state of the market. The truth-telling requirement, in particular, implies that for any pair of values β and β' in $[\underline{\beta}, \bar{\beta}]$, we have

$$t(\beta) + R(\beta, e(\beta, \beta)) - R(\beta, 0) \geq t(\beta') + R(\beta, e(\beta', \beta)) - R(\beta, 0) \tag{3.10}$$

$$t(\beta') + R(\beta', e(\beta', \beta')) - R(\beta^{'}, 0) \geq t(\beta) + R(\beta', e(\beta, \beta')) - R(\beta^{'}, 0) \tag{3.11}$$

.

The above two inequalities imply that the payoff when the publisher announces the true market state is always equal or higher than the payoff when the publisher announces any false market state. In other words, when the true market state is β, then the payoff if announcing β is equal or higher than the payoff if announcing $\beta^{'}$ (3.10). And when the true market state is $\beta^{'}$, then the payoff if announcing $\beta^{'}$ is equal or higher than the payoff if announcing β (3.11).

In the following paragraphs, we will simplify the truth-telling requirement to make it clearer for the subsequent development of the model.

Adding up (3.10) and (3.11) gives

$$R(\beta, e(\beta, \beta)) - R(\beta, e(\beta', \beta)) \geq R(\beta', e(\beta, \beta')) - R(\beta', e(\beta', \beta'))$$

By the definite integrating rule, the above condition can be restated as

$$\int_{\beta'}^{\beta} R_2(\beta, e(x, \beta))e_1(x, \beta)dx \geq \int_{\beta'}^{\beta} R_2(\beta', e(x, \beta'))e_1(x, \beta')$$

Again applying the definite integrating rule to the above inequality, we get

$$\int_{\beta'}^{\beta}\int_{\beta'}^{\beta}[R_{21}(y, e(x, y)) + R_{22}(y, e(x, y))e_2(x, y)]e_1(x, y) + R_2(y, e(x, y))e_{12}(x, y)dxdy \geq 0 \tag{3.12}$$

Given the assumptions that $R_{12}(\beta, e) = 0$, $R_{22}(\beta, e) < 0$, $R_2(\beta, e) < 0$, and the properties of equations $e_1(\hat{\beta}, \beta)$, $e_2(\hat{\beta}, \beta)$, $e_{12}(\hat{\beta}, \beta)$ based on the model setup, we have that the truth-telling requirement is identical to [3]

$$\hat{q}'(\hat{\beta}) \geq 0 \tag{3.13}$$

In order to ensure that announcing the true market state generates global maximization of expected payoff for the publisher, the mechanism must satisfy the first-order condition for truth-telling. The first-order condition for truth-telling implies that reporting the true state of the market satisfies the first-order-condition for maximizing the publisher's payoff over the choice range of the report. Equivalently, $\hat{\beta} = \beta$ maximizes $u_p(\beta, \hat{\beta})$. Thus, the first-order condition for truth-telling can be stated as

$$u_{p2}(\beta, \hat{\beta} = \beta) = 0 \tag{3.14}$$

[3] For the detailed proof, please see the Appendix C.2

where u_{p2} denote the derivative of the payoff function for the publisher w.r.t the announced market state (the second argument). So that the conditions (3.13) and (3.14) are the necessary conditions for truth-telling. [4]

Publisher's Marginal Rents Requirement

In this section, we discuss the required marginal rents for the publisher under the truth-telling condition. Let $U_p(\beta) \equiv u_p(\beta, \beta)$ denote the publisher's payoff (or rents) when the true market state is β under truth-telling requirement. Then (3.14) (or the envelope theorem applied to the maximization of (3.9)) yields

$$\begin{aligned} \dot{U}_p &= s_2^{'}(\beta) - p_2^{'}(e(\beta, \beta))[-\frac{s_1^{'}(\beta)}{p_1^{'}(e(\beta, \beta))}] - s_2^{'}(\beta) \\ &= p_2^{'}(e(\beta, \beta))[\frac{s_1^{'}(\beta)}{p_1^{'}(e(\beta, \beta))}] \end{aligned} \tag{3.15}$$

where $\dot{U}_p$ denotes the derivative of U_p w.r.t. the true market state, β, under truth-telling condition. We refer to the above function $\dot{U}_p(\beta)$ as the publisher's "rent speed" function as it measures how fast the publisher's rents increase as the market state β improves. Note that $\dot{U}_p$ is always positive for $\beta \in [\underline{\beta}, \bar{\beta}]$ according to the assumptions $s_1^{'}(\beta) > 0$,$p_1^{'}(e) > 0$ and $p_2^{'}(e) > 0$.

Under the truth-telling condition, we redefine effort as a function of true market state, β.

$$E(\beta) \equiv e(\hat{\beta} = \beta, \beta) \tag{3.16}$$

Then, the publisher's payoff can be restated as

$$U_p(\beta) = t(\beta) + R(\beta, E(\beta)) - R(\beta, 0) \tag{3.17}$$

[4] In the appendix (C.5), we proved that the necessary conditions for truth-telling (the simplified version of truth-telling requirement (3.13) and the first-order condition (3.14) for truth-telling) are the sufficient conditions for truth-telling as well.

3.3.2 The Traditional Advertiser's Problem

We now consider the traditional advertiser's problem with respect to maximizing his expected payoff by designing the optimal contract. Based on the publisher's reported market state $\hat{\beta}$, the contract returns an associated absolute impression quality requirement, $\hat{q}(\hat{\beta})$, for the publisher and also an associated guaranteed payment to the publisher, $t(\hat{\beta})$. As discussed in the previous section, given the absolute impression quality requirement, and under the truth-telling condition, the associated effort can be expressed as $E(\beta)$ (3.16). The traditional advertiser's objective function under truth-telling condition can be stated as

$$\max_{E(\cdot)} \int_{\underline{\beta}}^{\bar{\beta}} q(\beta, E(\beta)) - t(\beta) dF(\beta) \tag{3.18}$$

subject to truth-telling constraints $\hat{q}^{'}(\hat{\beta}) \geq 0$, and the marginal rent requirement, $\dot{U}_p = p_2^{'}(e(\beta,\beta))[\frac{s_1^{'}(\beta)}{p_1^{'}(e(\beta,\beta))}]$.

Rearranging the publisher's payoff function under truth-telling conditions, $U_p(\beta) = t(\beta) + R(\beta, E(\beta)) - R(\beta, 0)$, the monetary transfer can be stated as

$$t(\beta) = U_p(\beta) - R(\beta, E(\beta)) + R(\beta, 0) \tag{3.19}$$

Then, the advertiser's objective function can be restated as

$$\max_{E(\cdot)} \int_{\underline{\beta}}^{\bar{\beta}} q(\beta, E(\beta)) + R(\beta, E(\beta)) - U_p(\beta) - R(\beta, 0) dF(\beta) \tag{3.20}$$

subject to constraints

$$\dot{E}(\beta) \geq -\frac{q_1(\beta, e)}{q_2(\beta, e)}$$

[5], and

$$\dot{U}_p(\beta) = p_2^{'}(e(\beta,\beta))[\frac{s_1^{'}(\beta)}{p_1^{'}(e(\beta,\beta))}]$$

[5]Note the constraint $\dot{E}(\beta) \geq -\frac{q_1(\beta,e)}{q_2(\beta,e)}$ is the rewriting of the constraint $\hat{q}'(\hat{\beta}) \geq 0$ as $\dot{E}(\beta) = \frac{\hat{q}'}{q_2(\beta,e)} - \frac{q_1(\beta,e)}{q_2(\beta,e)}$.

Given that the publisher's "rent speed" function measures the marginal rents for the publisher as the true market state β improves, then integrating $\dot{U}_p$ over $[\underline{\beta}, \beta]$, we can rewrite the publisher's rents (or net payoff), $U_p(\beta)$, as

$$U_p(\beta) = \int_{\underline{\beta}}^{\beta} p_2'(e(\tilde{\beta}, \tilde{\beta}))[\frac{s_1'(\tilde{\beta})}{p_1'(e(\tilde{\beta}, \tilde{\beta}))}]d\tilde{\beta} + U(\underline{\beta}) \tag{3.21}$$

Since the publisher's rents are costly for the advertiser, the traditional advertiser would set $U(\underline{\beta}) = 0$. Then, according to integrating by parts rule, the expected rents transferred by the advertiser to the publisher is

$$\int_{\underline{\beta}}^{\bar{\beta}} U_p(\beta) dF(\beta) = \int_{\underline{\beta}}^{\bar{\beta}} \frac{1 - F(\beta)}{f(\beta)} [p_2'(E(\beta)) \frac{s_1'(\beta)}{p_1'(E(\beta))}] dF(\beta) \tag{3.22}$$

. [6]

Substituting the expected rents given up by the traditional advertiser into the advertiser's objective function, the advertiser's objective function can be rewritten as

$$\max_{E(\cdot)} \int_{\underline{\beta}}^{\bar{\beta}} \left\{ q(\beta, E(\beta)) + R(\beta, E(\beta)) - \frac{1 - F(\beta)}{f(\beta)} [p_2'(E(\beta)) \frac{s_1'(\beta)}{p_1'(E(\beta))}] - R(\beta, 0) \right\} dF(\beta) \tag{3.23}$$

subject to the constraint

$$\dot{E}(\beta) \geq -\frac{q_1(\beta, e)}{q_2(\beta, e)} \tag{3.24}$$

Equilibrium for the Optimal Contract

In this subsection, we identify the equilibrium of the contract and discuss some properties at equilibrium. Ignoring the constraint (3.24) for the moment, maximizing the advertiser's objective function w.r.t the effort function $E(\cdot)$, we find

[6] For detailed proof, please refer to Appendix C.6.

$$p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta)) = \frac{1-F(\beta)}{f(\beta)}[s_1^{'}(\beta)\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p_1^{'}(E^*(\beta)))^2}] \tag{3.25}$$

We will later show that the truth revelation condition is naturally satisfied at the optimal effort level.

The above equation implicitly defines the effort level, $E^*(\beta)$, at equilibrium. In other words, under the truth-telling conditions, $E^*(\beta)$ is the optimal effort level the advertiser wants to induce from the publisher by the design of the absolute impression quality $\hat{q}(\hat{\beta})$ and monetary transfer $t(\hat{\beta})$, with respect to maximizing his expected payoff.

Comparative Statics of the Effort Level at Equilibrium

In this subsection, we discuss some interesting comparative statics of the effort level at equilibrium. Differentiating the equilibrium effort level equation (3.25) w.r.t to the market state β, we get

$$\dot{E}^*(\beta) = \frac{dE^*}{d\beta} = \frac{\frac{d}{d\beta}[\frac{1-F(\beta)}{f(\beta)}]s_1^{'}(\beta) + \frac{1-F(\beta)}{f(\beta)}s_1^{''}(\beta)\}[\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p_1^{'}(E^*(\beta)))^2}]}{p_1^{''}(E^*(\beta)) - p_2^{''}(E^*(\beta)) - \frac{1-F(\beta)}{f(\beta)}s_1^{'}(\beta)\frac{d^2}{dE^{*2}}[\frac{p_2^{'}(E^*(\beta))}{p_1^{'}(E^*(\beta))}]} \tag{3.26}$$

where $\dot{E}^*(\beta)$ represents the marginal change in the induced effort at equilibrium as the realized market state β improves. [7]

With the following two assumptions, we have proposition 3.1.

Assumption 1. $\frac{d}{d\beta}\frac{1-F(\beta)}{f(\beta)} < 0$, *on the support of* $[\underline{\beta}, \bar{\beta}]$. *It is also known as monotone hazard rate assumption, which is satisfied by most usual distributions and is standard in contract theory.*

and that

Assumption 2. $\frac{d^2}{dE^{*2}}[\frac{p_2^{'}(E^*(\beta))}{p_1^{'}(E^*(\beta))}] \geq 0$. *This is equivalent to assume that* $\frac{d^2}{dE^{*2}}[-\frac{R_2(\beta,e)}{q_2(\beta,e)}] \geq 0$.

[7]For detailed derivative of (3.26), please refer to Appendix C.7.

Assumption 2 might look abstract to understand. We analyze the assumption with a simple example. Suppose the spot market revenue takes the form of $R(\beta, e) = a\beta - re^c$, and the impression quality takes the form of $q(\beta, e) = b\beta + ve^d$. Then the above assumption is equivalent to assume that $c - d \geq 1$. For example, $R(\beta, e) = \beta - e^2$ and $q(\beta, e) = \beta + e^{\frac{1}{2}}$ satisfy the above assumption. This assumption captures a broad range of reasonable revenue and impression quality functions. Now we can state the comparative static in the following proposition.

Proposition 3.1. *Under the truth-telling conditions, the equilibrium effort level, $E^*(\beta)$, exerted by the publisher induced by the contracting mechanism is increasing in the true market state, β. What's more, the expected rents for the publisher, $U_p(\beta)$, is increasing in the market state, β as well. (For detailed proof, please refer to Appendix C.8.)*

Proposition 3.1 implies that at any interior equilibrium, we have $\dot{E}^*(\beta) > 0$. And naturally the publisher's constraint (3.24) is satisfied at equilibrium[8]. In other words, it is saying that the contract is designed such that at equilibrium the better the state of the market, the greater effort is induced by the contract. And according to marginal rents function (3.15) under truth-telling condition, the rents to the publisher are increasing as the market state β improves.

Mechanism of the Contract

Next, we are going to identify the associated absolute impression quality requirement, $\hat{q}(\hat{\beta})$, and the monetary transfer, $t(\hat{\beta})$, for the contract.

Given the publisher's rent function $U_p(\beta) = \int_{\underline{\beta}}^{\beta} R_1(\tilde{\beta}, E^*(\tilde{\beta})) + R_2(\tilde{\beta}, E^*(\tilde{\beta}))e_2(\tilde{\beta}, \tilde{\beta})d\tilde{\beta}$, and the effort level $E^*(\beta)$ implicitly defined by the equation (3.25), we have the monetary transfer as follows

$$\begin{aligned} t(\beta) &= U_p(\beta) - R(\beta, E^*(\beta)) + R(\beta, 0) \\ &= \int_{\underline{\beta}}^{\beta} R_2(\tilde{\beta}, E^*(\tilde{\beta}))e_2(\tilde{\beta}, \tilde{\beta})d\tilde{\beta} - R(\beta, E^*(\beta)) + R(\beta, 0) \end{aligned} \tag{3.27}$$

[8] $\dot{E}(\beta) > 0 > -\frac{q_1(\beta,e)}{q_2(\beta,e)}$

Next, given that the publisher's effort must satisfy $\hat{q}(\hat{\beta}) = q(\beta, e)$ under the truth-telling conditions, the required impression quality is

$$\hat{q}(\beta) = q(\beta, E^*(\beta)) \tag{3.28}$$

when the realized market state is β.

We summarize the absolute impression quality requirement and the associated guaranteed payment of the incentive-compatible contract mechanism in the following proposition.

Proposition 3.2. *When the publisher announces market state $\hat{\beta}$, the required absolute impression quality is defined as $\hat{q}(\hat{\beta}) = q(\hat{\beta}, E^*(\hat{\beta}))$; The monetary transfer the traditional contractor makes to the publisher is defined as $t(\hat{\beta}) = \int_{\underline{\beta}}^{\hat{\beta}} p_2'(e(\tilde{\beta}, \tilde{\beta}))[\frac{s_1'(\tilde{\beta})}{p_1'(e(\tilde{\beta},\tilde{\beta}))}]d\tilde{\beta} - R(\hat{\beta}, E^*(\hat{\beta})) + R(\hat{\beta}, 0)$.*

3.4 Correlated Markets

We now consider the case where the contract utilizes the price information from the spot market. We assume that the realized bidding price unbiasedly reflects the true quality of the impressions allocated to the spot market. Thus, the fluctuation of the bidding price in the spot market reflects the fluctuation of impression quality that is impacted by the market state β and also the publisher's effort e, that she devotes to the traditional contractor, together with random variation in advertising demand. In general, the better the market state β, the higher the expected price in the spot market; and that the more effort e the publisher exerts to the traditional contractor, the lower the expected price in the spot market.

We use α to denote the aggregate real-time bidding price level in the spot market. It is assumed that α and β follow the joint distribution probability density function $f_{\alpha,\beta}(\alpha, \beta)$ on the support of $[\underline{\beta}, \bar{\beta}]$. $f_c(\alpha|\beta)$ refers to the conditional probability of α equal to a specific α given β equals to a specific β. Naturally $F_c(\alpha|\beta)$ refers to the conditional cumulative probability of α smaller or equal to a specific α given β equals to a specific β. When there is positive effort e the publisher make to the traditional contractor, it also impacts the conditional distribution according to $f_c(\alpha|\beta, e)$ and $F_c(\alpha|\beta, e)$. To make this more concrete, consider the following example relating α, β and the effort e. β follows a Beta distribution over the support $[\underline{\beta}, \bar{\beta}]$. α follows a distribution, whose mean is around the realization of β on the support of $[\underline{\alpha}, \bar{\alpha}]$. Then a positive effort e is shifting the mean of α's distribution away from the realization of β to the left. Thus, we have $\frac{d}{d\beta}(F_c(\alpha|\beta, e)) < 0$ and $\frac{d}{de}(F_c(\alpha|\beta, e)) > 0$.

Let $t(\hat{\beta}) + m(\hat{\beta}, \alpha)$, $\hat{q}(\hat{\beta})$ be a revelation mechanism, where $\hat{\beta}$ denotes the announcement of the market state by the publisher. That is if the publisher announces $\hat{\beta}$, she must realize a absolute overall quality of impressions allocated to traditional contractor, $\hat{q}$, and will receive a guaranteed transfer $t(\hat{\beta})$ from the traditional contractor, and a lottery transfer $m(\hat{\beta}, \alpha)$. The lottery works as follows. If the realized spot market price level is lower or equal to a threshold α^*, then a reward $\bar{m}(\hat{\beta})$ as a function of the announced market state $\hat{\beta}$ will be transferred to the publisher; if the realized spot market price level is greater than the threshold α^*, then there is no reward, $\underline{m}(\hat{\beta}) = 0$.

The publisher's expected payoff as a function the realized market state β , the announced one $\hat{\beta}$, and the effort level e is

$$u_p(\beta, \hat{\beta}, e) \equiv t(\hat{\beta}) + \bar{m}(\hat{\beta})F_c(\alpha^*|\beta, e) - (R(\beta, 0)) - R(\beta, e)) \tag{3.29}$$

That is the guaranteed transfer plus the expected lottery transfer the publisher received from the traditional contractor, minus the forgone spot market expected revenue due to the effort she exerted to the traditional contractor.

The spot market expected revenue function, $R(\beta, e)$, can be understood in the following way. Normalizing the expected revenue from spot market to be the expected price level in the spot market, then $R(\beta, e) = \int_{\underline{\alpha}}^{\bar{\alpha}} f_c(\alpha|\beta, e)\alpha d\alpha$, the mean of the spot market price distribution. $R(\beta, 0)$ represents the revenue the publisher could have earn if there were no traditional contract market. Thus, $R(\beta, 0) - R(\beta, e)$ denotes the publisher's opportunity cost of exerting e effort level to the traditional contract. And consistently with the model set-up assumptions, we still have $R_1(\beta, e) > 0$, $R_2(\beta, e) < 0$ and $R_{22}(\beta, e) < 0$. The last assumption $R_{22}(\beta, e) < 0$ is saying that as the publisher exerts more and more effort to the traditional contractor, it is shifting the spot market price distribution mean to the left at an increasing speed. Thus, the expected revenue the publisher gets from the spot market is decreasing at an increasing speed as she devotes increasing effort to the traditional contractor. The traditional advertiser's expected payoff as a function of realized market state, β, the announced one, $\hat{\beta}$, the effort by the publisher, e, and the realized spot market price, α, is defined as

$$u_a(\beta, \hat{\beta}, e) \equiv q(\beta, e) - t(\hat{\beta}) - \bar{m}(\hat{\beta}) F_c(\alpha^* | \beta, e) \tag{3.30}$$

That is the quality of the impressions the traditional contractor received minus the guaranteed transfer and minus the lottery transfer as the monetary return from the impressions is normalized to be the quality of those impressions.

3.4.1 Publisher's Problem

Keeping all the notations consistent with the model set-up, the publisher's expected payoff as a function of the realized market state, β, the announced one, $\hat{\beta}$, the realized spot market price, α, and the effort level the publisher puts into the traditional contract, e, is

$$u_p(\beta, \hat{\beta}) \equiv t(\hat{\beta}) + \bar{m}(\hat{\beta}) F_c(\alpha^* | \beta, e(\hat{\beta}, \beta)) + R(\beta, e(\hat{\beta}, \beta)) - R(\beta, 0) \tag{3.31}$$

where $e(\hat{\beta}, \beta)$ is the effort level the publisher has to exert if she announces $\hat{\beta}$ while the true market state is β.

Truth Revelation Requirement

The truth-telling requirement for any pair of values β and β' in $[\underline{\beta}, \bar{\beta}]$ is analogous to the benchmark case truth-telling conditions (3.10) and (3.11).

$$t(\beta) + \bar{m}(\beta) F_c(\alpha^* | \beta, e(\beta, \beta)) + R(\beta, e(\beta, \beta)) \geq t(\beta') + \bar{m}(\beta') F_c(\alpha^* | \beta, e(\beta', \beta)) + R(\beta, e(\beta', \beta)) \tag{3.32}$$

$$t(\beta')+\bar{m}(\beta')F_c(\alpha^*|\beta', e(\beta',\beta'))+R(\beta', e(\beta',\beta')) \geq t(\beta)+\bar{m}(\beta)F_c(\alpha^*|\beta', e(\beta,\beta'))+R(\beta', e(\beta,\beta')) \tag{3.33}$$

By rearranging and simplifying the above inequalities, the truth-telling requirement reduces to

$$\begin{aligned}&\int_{\beta'}^{\beta}\int_{\beta'}^{\beta} R_{22}(y, e(x,y))e_2(x,y)e_1(x,y) + R_2(y, e(x,y))e_{12}(x,y)\\ &+ \frac{d}{dx}[\bar{m}(x)[\frac{d}{d\beta}F_c(\alpha^*|y, e(x,y)) + \frac{d}{de}F_c(\alpha^*|y, e(x,y))e_2(x,y)]]dxdy \geq 0\end{aligned}$$

.

With the above inequality condition, it is identical to have the following constraint.

$$\begin{aligned}&R_{22}(y, e(x,y))e_2(x,y)e_1(x,y) + R_2(y, e(x,y))e_{12}(x,y)\\ &+ \frac{d}{dx}[\bar{m}(x)[\frac{d}{d\beta}F_c(\alpha^*|y, e(x,y)) + \frac{d}{de}F_c(\alpha^*|y, e(x,y))e_2(x,y)]] \geq 0\end{aligned} \tag{3.34}$$

The above inequality is the simplified condition, which is equivalent to the truth-telling requirement. (For detailed proof, please refer to the Appendix C.9.) In the remainder of this section, the truth-telling requirement refers to the above condition.

The truth-telling requirement also implies that announcing the true market situation ($\hat{\beta} = \beta$) maximizes the publisher's expected payoff $u_p(\beta, \hat{\beta})$. The first-order condition under truth-telling condition can be written as

$$u_{p2}(\beta, \hat{\beta} = \beta) = 0 \tag{3.35}$$

The Appendix C.10 shows that if the truth-telling condition (3.34) and the first-order condition (3.35) for truth-telling are satisfied, these necessary conditions are sufficient.

Marginal Rents Requirement

In this section, we discuss the design of the lottery reward and the associated marginal rents for the publisher under truth-telling conditions. Let's consider the lottery reward magnitude as follows.

$$\bar{m}(\beta) = -\frac{R_2(\beta, e(\beta,\beta))e_2(\beta,\beta) - \phi(\beta)}{\frac{d}{d\beta}(F_c(\alpha^*|\beta, e(\beta,\beta))) + \frac{d}{de}(F_c(\alpha^*|\beta, e(\beta,\beta))e_2(\beta,\beta))} \tag{3.36}$$

where $\phi(\cdot)$ is a function of the announced type $\hat{\beta}$, but under truth-telling conditions it shows as $\phi(\beta)$. With such a design of the reward magnitude, we have that the function $\phi(\beta)$ is equivalent to the marginal rents for the publisher under the truth-telling first-order condition. [9] Consequently, by designing the function form $\phi(\cdot)$, the advertiser can control the publisher's marginal rents as the market state improves, $\dot{U}(\beta)$.

What's more, it is easy to show that with some rearrangement of the equation (3.36), the truth-telling requirement (3.34) becomes [10]

$$\frac{d}{dx}\phi(x) \geq 0 \tag{3.37}$$

Similar to the isolated market case, with the absolute impression quality requirement function, $\hat{q}(\hat{\beta})$, the realized market state, β, and the announced market state, $\hat{\beta}$, there is an associated effort level the publisher must exert in order to satisfy the requirement, $e(\hat{\beta}, \beta)$. For clear notation, we redefine effort level under truth-telling as a function of realized market state β when $\hat{\beta} = \beta$, as

$$E(\beta) \equiv e(\hat{\beta} = \beta, \beta)$$

Substituting the effort function above into the publisher's objective function, we restate the publisher's objective function as

$$U_p(\beta) = t(\beta) + \bar{m}(\beta)F_c(\alpha^*|\beta, E(\beta)) + R(\beta, E(\beta)) - R(\beta, 0)$$

[9] For a detailed proof, please refer to the Appendix C.11

[10] For a detailed derivation, please refer to Appendix C.12.

3.4.2 The Advertiser's Problem

Now we consider the traditional advertiser's problem of maximizing his expected payoff, which is the monetary return from the impressions delivered by the publisher, $q(\cdot)$, minus the guaranteed payment to the publisher, $t(\cdot)$, minus the expected reward paid to the publisher. The traditional advertiser's objective function is stated as

$$\max_{E(\cdot),\bar{m}(\cdot)} \int_{\underline{\beta}}^{\bar{\beta}} q(\beta, E(\beta)) - t(\beta) - \bar{m}(\beta)F_c(\alpha^*|\beta, E(\cdot))dF(\beta)$$

Rearranging the publisher's expected payoff (rents) function under truth-telling condition, we get

$$-t(\beta) - \bar{m}(\beta)F_c(\alpha^*|\beta, E(\beta)) = R(\beta, E(\beta)) - R(\beta, 0) - U_p(\beta)$$

Substituting the above equation into the traditional advertiser's objective function, we have

$$\max_{E(\cdot),\bar{m}(\beta)} \int_{\underline{\beta}}^{\bar{\beta}} q(\beta, E(\beta)) + R(\beta, E(\beta)) - R(\beta, 0) - U_p(\beta)dF(\beta)$$

subject to truth-telling requirement, $\frac{d}{d}\phi(x) \geq 0$ (3.37).

and the marginal rents requirement, derived from the truth-telling first-order condition, (3.35).

$$\begin{aligned}\dot{U}_p(\beta) = \bar{m}(\beta)[&\frac{d}{d\beta}(F_c(\alpha^*|\beta, E(\beta)) + \frac{d}{de}(F_c(\alpha^*|\beta, E(\beta))e_2(\beta, \beta))] \\ &+ R_1(\beta, E(\beta)) + R_2(\beta, E(\beta))e_2(\beta, \beta) + R_1(\beta, 0) = \phi(\beta)\end{aligned}$$

By integrating the above marginal rents function over $[\underline{\beta}, \beta]$, we can rewrite the expected rents for the publisher contingent on the realized market state, $U_p(\beta)$, as the following

$$U_p(\beta) = \int_{\underline{\beta}}^{\beta} \phi(\tilde{\beta})d\tilde{\beta} + U_p(\underline{\beta})$$

Since the publisher's rents are costly for advertiser given $\phi(\beta) \geq 0$, the advertise will set $U_p(\underline{\beta}) = 0$. To ensure that the publisher's participation constraint is satisfied, we also have $\phi(\beta) \geq 0$. In other words, the publisher's expected rents must be non-negative. So we have the expected rents for the publisher under truth-telling condition when true market situation is β, as

$$U_p(\beta) = \int_{\underline{\beta}}^{\beta} \phi(\tilde{\beta}) d\tilde{\beta} \tag{3.38}$$

Integrating the above publisher's expected rents function over the market state distribution, we have the expected rents for the publisher based on the market state distribution, as

$$\begin{aligned}\int_{\underline{\beta}}^{\bar{\beta}} U_p(\beta) &= \int_{\underline{\beta}}^{\bar{\beta}} \int_{\underline{\beta}}^{\beta} \phi(\tilde{\beta}) d\tilde{\beta} d\beta \\ &= \int_{\underline{\beta}}^{\bar{\beta}} \frac{1}{f(\beta)} \int_{\underline{\beta}}^{\beta} \phi(\tilde{\beta}) d\tilde{\beta} dF(\beta)\end{aligned}$$

Substituting the expected rents which the traditional advertiser must transfer to the publisher into the advertiser's objective function, the advertiser's objective function can be restated as

$$\max_{E(\cdot)} \int_{\underline{\beta}}^{\bar{\beta}} q(\beta, E(\beta)) + R(\beta, E(\beta)) - R(\beta, 0) - (\frac{1}{f(\beta)} \int_{\underline{\beta}}^{\beta} \phi(\tilde{\beta}) d\tilde{\beta}) dF(\beta) \tag{3.39}$$

subject to the truth-telling requirement, $\frac{d}{dx}\phi(x) \geq 0$ (3.37).

The Equilibrium of the Optimal Contract

Ignore the truth-telling requirement (3.37) for the moment, by differentiating the advertiser' objective function w.r.t the effort level, $E(\cdot)$, we find that at the optimum, the advertiser should design the contract mechanism such that the induced effort level $E(\beta)$ is implicitly defined as

$$q_2(\beta, E^*(\beta)) + R_2(\beta, E^*(\beta)) = 0 \tag{3.40}$$

Recall the equilibrium for the social efficiency case (3.4), as discussed in the section 3.2, the equilibrium effort level achieved in this case is almost the same as in the social efficiency case. Similar to the social efficiency case, the induced effort level in this contract, E^* is independent of the realized market state, β.

$$\dot{E}^*(\beta) = \frac{dE^*}{d\beta} = 0 \tag{3.41}$$

where $\dot{E}^*(\beta)$ denotes the derivative of $E^*(\beta)$ w.r.t. β.

What's more, the rents the advertiser pays to the publisher are $U_p(\beta) = \int_{\underline{\beta}}^{\beta} \phi(\tilde{\beta})d\tilde{\beta}$ (3.38). Thus, by setting $\phi(\cdot)$ arbitrarily small but positive with $\frac{d}{dx}\phi(x) \geq 0$, the advertiser can extract the rest rents, which is almost the full rents. When the traditional advertiser sets $\phi(x) = 0$, then the traditional extracts the full rents and the publisher gets zero expected rents. The publisher's participation constraint is just binding in such case. We state the above discussion in the following proposition.

Proposition 3.3. *At equilibrium, the induced effort level $E^*(\beta)$ remains unchanged as the market state, β, changes. The rents left to the publisher is defined by $\int_{\underline{\beta}}^{\beta} \phi(\tilde{\beta})d\tilde{\beta}$. The publisher's rents are weakly increasing in the market state, β, but are arbitrarily small as the traditional advertiser has the incentive to set $\phi(\hat{\beta})$ arbitrarily small (closing to zero) by designing the reward magnitude in the lottery in order to maximize her own expected payoff.*

Optimal Mechanism of the Contract

In this subsection, we discuss the optimal mechanism of such a contract.

As long as $\frac{d}{d}\phi(x) \geq 0$, the publisher's truth-telling constraint is satisfied, the advertiser would set the magnitude of the reward as

$$\bar{m}(\beta) = -\frac{R_1(\beta, e(\beta, \beta))) + R_2(\beta, e(\beta, \beta))e_2(\beta, \beta) + R_1(\beta, 0) - \phi(\beta)}{\frac{d}{d\beta}(F_c(\alpha^*|\beta, e(\beta, \beta))) + \frac{d}{de}(F_c(\alpha^*|\beta, e(\beta, \beta))e_2(\beta, \beta))}, \tag{3.42}$$

Now let's define the guaranteed transfer the traditional contractor makes to the publisher, $t(\hat{\beta})$. Recall the publisher's objective function under truth-telling conditions

$$U_p(\beta) = t(\beta) + \bar{m}(\beta)F_c(\alpha^*|\beta, E^*(\beta)) + R(\beta, E^*(\beta)) - R(\beta, 0) \tag{3.43}$$

Substituting the publisher's rents under truth-telling conditions, $U_p(\beta) = \int_{\underline{\beta}}^{\beta} \phi(\tilde{\beta})d\tilde{\beta}$, into the above equation, then we have

$$t(\hat{\beta}) = \int_{\underline{\beta}}^{\hat{\beta}} \phi(\tilde{\beta})d\tilde{\beta} - \bar{m}(\hat{\beta})F_c(\alpha^*|\hat{\beta}, E^*(\hat{\beta})) - R(\hat{\beta}, E^*(\hat{\beta})) + R(\hat{\beta}, 0) \tag{3.44}$$

Now, let's define the absolute impression requirement when the publisher announces $\hat{\beta}$. The absolute impression quality requirement as a function of the announced market state is set as

$$\hat{q}(\hat{\beta}) = (\hat{\beta}, E^*(\hat{\beta})) \tag{3.45}$$

where $E^*(\cdot)$ is implicitly defined according to the following condition

$$q_2(\beta, E^*(\beta)) + R_2(\beta, E^*(\beta)) = 0 \tag{3.46}$$

Summarizing the details of the contracting mechanism in the following proposition, we have

Proposition 3.4. *When the publisher announces the market state, $\hat{\beta}$, the required absolute impression quality is defined as $\hat{q}(\hat{\beta}) = q(\hat{\beta}, E^*(\hat{\beta}))$; The guaranteed transfer the traditional contractor makes to the publisher is defined as $t(\hat{\beta}) = \int_{\underline{\beta}}^{\hat{\beta}} \phi(\tilde{\beta})d\tilde{\beta} - \bar{m}(\hat{\beta})F_c(\alpha^*|\hat{\beta}, E^*(\hat{\beta})) - R(\hat{\beta}, E^*(\hat{\beta})) + R(\hat{\beta}, 0)$; The magnitude of reward of the lottery is defined as $\bar{m}(\hat{\beta}) = -\frac{R_2(\hat{\beta}, e(\hat{\beta},\hat{\beta}))e_2(\hat{\beta},\hat{\beta}) - \phi(\hat{\beta})}{\frac{d}{d\hat{\beta}}(F_c(\alpha^*|\hat{\beta}, e(\hat{\beta},\hat{\beta}))) + \frac{d}{de}(F_c(\alpha^*|\hat{\beta}, e(\hat{\beta},\hat{\beta}))e_2(\hat{\beta},\hat{\beta}))}$ when the threshold of the spot market price is set to be α^*.*

3.5 An Numerical Example Of The Two Contract Mechanism

In this section, we use a simplified numerical example to show how a contract utilizing pricing information from the spot market increases the efficiency level regarding the effort exerted by the publisher and the traditional advertiser's expected payoff.

Define the realized impression quality function (the guaranteed contractor) as

$$q(\beta, e) = \beta + e^{\frac{1}{2}} \tag{3.47}$$

The expected revenue from the spot market is defined as

$$R(\beta, e) = \beta - e^2 \tag{3.48}$$

The market situation, β, follows a Beta distribution on the normalized support of $[0, 1]$, with the two shape parameters as $a = 3$ and $b = 3$. This is a symmetric distribution around the mode and also the mean at $\frac{1}{2}$ on a bounded range.

3.5.1 The Socially Efficient Scenario

Let's first consider the socially efficient case. The social planner's objective is to maximize the total surplus in the market, the sum of the traditional advertiser's and the publisher's expected payoff. So we define the social planner's objective as

$$\begin{aligned} \max_{e} \quad & q(\beta, e) + R(\beta, e) \\ & \beta + e^{\frac{1}{2}} + \beta - e^2 \end{aligned} \tag{3.49}$$

Maximizing the social planner's objective function w.r.t. the effort level, we get the F.O.C

$$\frac{1}{2} e^{**-\frac{1}{2}} - 2e^{**} = 0 \tag{3.50}$$

Dividing both sides with $e^{**\frac{1}{2}}$, we have

$$\frac{1}{2} e^{**-1} - 2e^{**\frac{1}{2}} = 0 \tag{3.51}$$

Solving the above equation, we get

$$e^{**} = 0.4 \tag{3.52}$$

We can see that the socially efficient level of effort is independent of the realized market state. In other words, by the construction of our model, no matter what the true market situation is, it is optimal to have a certain amount of effort devoted to the traditional contractor such that the relative impression quality between the two markets is fixed even though the absolute quality might be dependent on the realized market state.

3.5.2 Isolated Market

We now consider the contracting mechanism that doesn't utilize the pricing information from the spot market. Such contract work as follows. As only the publisher observes the true market state, β, and the traditional advertiser only knows the distribution of market state, a Beta distribution on the support of $[0, 1]$. Based on the publisher's announced market state, $\hat{\beta}$, the traditional advertiser sets the impression quality requirement, $\hat{q}(\hat{\beta})$, as a function of the announced market state, that the publisher has to meet. And the traditional advertiser makes a guaranteed monetary transfer, $t(\hat{\beta})$, as a function of announced market state, to the publisher. According to the contract mechanism in the isolated market, the traditional contractor would set the impression quality requirement as

$$\hat{q}(\hat{\beta}) = \hat{\beta} + \Theta(\hat{\beta})^{\frac{1}{2}} \tag{3.53}$$

and the associated guaranteed transfer as

$$t(\hat{\beta}) = \int_{\underline{\beta}}^{\hat{\beta}} 4\Theta^{\frac{3}{2}}(\tilde{\beta}, \tilde{\beta})d\tilde{\beta} + \Theta(\hat{\beta})^2 \tag{3.54}$$

where

$$\Theta(\hat{\beta}) = \{0.5\{1 - 8(\frac{1 - F(\hat{\beta})}{f(\hat{\beta})})^3 + \sqrt{-16(\frac{1 - F(\hat{\beta})}{f(\hat{\beta})})^{\frac{1}{3}} + 1}\}^{\frac{1}{3}}$$
$$+ \frac{2(\frac{1-F(\hat{\beta})}{f(\hat{\beta})})^2}{\{1 - 8(\frac{1-F(\hat{\beta})}{f(\hat{\beta})})^3 + \sqrt{-16(\frac{1-F(\hat{\beta})}{f(\hat{\beta})})^{\frac{1}{3}} + 1}\}^{\frac{1}{3}}} - \frac{1 - F(\hat{\beta})}{f(\hat{\beta})}\}^2$$

.

Then, with the impression quality requirement, when the publisher announces market situation $\hat{\beta}$ but the true market situation is β, the effort level the publisher has to exert is determined by

$$\hat{\beta} + \Theta(\hat{\beta})^{\frac{1}{2}} = \beta + e^{\frac{1}{2}}$$

Rearranging the above equation, we get the effort level must be exerted by the publisher given the true and announced market state.

$$e(\hat{\beta}, \beta) = [\hat{\beta} - \beta + \Theta(\hat{\beta})^{\frac{1}{2}}]^2$$

Given the contracting mechanism, the publisher's objective function can be stated as

$$\begin{aligned}
\max_{\hat{\beta}} u_p(\hat{\beta}) &= t(\hat{\beta}) + R(\beta, e(\hat{\beta}, \beta) - R(\beta, 0) \\
&= \int_{\underline{\beta}}^{\hat{\beta}} 4\Theta^{\frac{3}{2}}(\tilde{\beta}, \tilde{\beta}) d\tilde{\beta} + \Theta^2(\hat{\beta}) + \beta - e(\hat{\beta})^2 - \beta \\
&= \int_{\underline{\beta}}^{\hat{\beta}} 4\Theta^{\frac{3}{2}}(\tilde{\beta}, \tilde{\beta}) d\tilde{\beta} + \Theta^2(\hat{\beta}) - e(\hat{\beta})^2 \\
&= \int_{\underline{\beta}}^{\hat{\beta}} 4\Theta^{\frac{3}{2}}(\tilde{\beta}, \tilde{\beta}) d\tilde{\beta} + \Theta(\hat{\beta})^2 - [\hat{\beta} - \beta + \Theta(\hat{\beta})^{\frac{1}{2}}]^4
\end{aligned}$$

Maximizing the publisher's objective function w.r.t the announced market state $\hat{\beta}$, we have the F.O.C

$$4\Theta(\hat{\beta})^{\frac{3}{2}} + 2\Theta(\hat{\beta})\frac{d\Theta}{d\hat{\beta}} = 4[\hat{\beta} - \beta + \Theta(\hat{\beta})^{\frac{1}{2}}]^3(1 + \frac{1}{2}\Theta(\hat{\beta})^{-\frac{1}{2}}\frac{d\Theta}{d\hat{\beta}}) \tag{3.55}$$

It is easy to verify that when $\hat{\beta} = \beta$, the F.O.C holds. Thus, it is the publisher's best interest to announce the true market state. And according to the effort function $e(\hat{\beta}) = [\hat{\beta} - \beta + \Theta(\hat{\beta})^{\frac{1}{2}}]^2$, the publisher would exert the effort level, when announces the true market situation, as

$$e^*(\beta, \beta) = \Theta(\beta)$$

Or equivalently,

$$\begin{aligned} e^*(\beta) = \Big\{0.5\Big\{1 - 8(\frac{1-F(\beta)}{f(\beta)})^3 + \sqrt{-16(\frac{1-F(\beta)}{f(\beta)})^{\frac{1}{3}} + 1}\Big\}^{\frac{1}{3}} \\ + \frac{2(\frac{1-F(\beta)}{f(\beta)})^2}{\Big\{1 - 8(\frac{1-F(\beta)}{f(\beta)})^3 + \sqrt{-16(\frac{1-F(\beta)}{f(\beta)})^{\frac{1}{3}} + 1}\Big\}^{\frac{1}{3}}} - \frac{1-F(\beta)}{f(\beta)}\Big\}^2 \end{aligned} \tag{3.56}$$

The above analysis shows that when taking into the consideration of the spot market expected revenue the publisher could have earned if not exerting effort to the traditional contractor (her opportunity cost of exerting effort to the traditional contractor), the guaranteed payment function is designed in such a way that it is the publisher's best interest to announce the true market situation, then satisfy the associated required impression quality target, and then, earn the associated guaranteed payment specified by the contract.

Relationship Between The Induced Effort and The Market State

Now, let's consider relationship between the induced effort level and the realized market situation, β. Rearranging the equilibrium effort function (3.56), we get

$$\frac{1}{2}e^{*-1} - 2e^{*\frac{1}{2}} = 6\frac{1-F(\beta)}{f(\beta)} \tag{3.57}$$

Differentiating the left-hand term w.r.t. e^*, we can show that the left-hand side is decreasing in the effort level e^*. And we have the assumption, $\frac{d}{d\beta}(\frac{1-F(\beta)}{f(\beta)}) < 0$. So, the equilibrium effort level e^* is increasing in the market state β. To identify the effort level when the market is in extreme scenario, let's consider the case when $\beta = \bar{\beta} = 1$. According to the equilibrium effort (3.56), the induced effort level is

$$\begin{aligned} e^*(\bar{\beta}) &= \left\{0.5 \times (1+1)^{\frac{1}{3}} + 0 - 0\right\}^2 \\ &= 0.40 \end{aligned}$$

When the true market state is at its highest possible level, $\bar{\beta} = 1$, the induced effort level is the same as the socially efficient level (section 3.5.1).

Now let's consider the case when the market staty is in its worst scenario, $\beta = \underline{\beta} = 0$. Then, the induced effort level is determined by

$$\frac{1}{2}e^{*-1} - 2e^{*\frac{1}{2}} = 6\frac{1-F(0)}{f(0)}$$

The right-hand side is positive infinity as the density function is approaching zero. Given the left-hand side is monotonically decreasing in the effort level, and that as the effort level approaches zero from the right side (the positive side), the term $\frac{1}{2}e^{*-1} - 2e^{*\frac{1}{2}}$ is going positive infinity, the induced effort level is $e^*(\underline{\beta}) = 0$. We have the example Figure 3.1 to illustrate the relationship between the realized market state and the publisher's induced effort from the contract at equilibrium.

In Figure 3.1, the market state variable β follows the Beta distribution with both shape parameter a and b equal to 3 on the support of $[0, 1]$. We confirm the above discussion by the example graph. As the market state approaches its worst scenario ($\beta = 0$), the induced effort is approaching zero; The induced effort is monotonically increasing with the market state; When the market state approaches it best scenario ($\beta = 1$), the induced effort is approaching the socially efficiently level 0.4.

Relationship Between Rents for the Publisher and Market State

Recall the publisher's expected payoff function, taking derivative of the publisher's expected payoff function w.r.t the realized market state, we have

$$\begin{aligned}\frac{d}{d\beta}u_p(\hat{\beta},\beta) &= R_1(\beta, e(\hat{\beta},\beta)) + R_2(\beta, e(\hat{\beta},\beta))e_2(\hat{\beta},\beta) - R_1(\beta,0)\\ &= R_2(\beta, e(\hat{\beta},\beta))e_2(\hat{\beta},\beta)\\ &= -2(\hat{\beta}-\beta+\Theta(\hat{\beta})^{\frac{1}{2}})^2[-2(\hat{\beta}-\beta+\Theta(\hat{\beta})^{\frac{1}{2}})]\\ &= 4(\hat{\beta}-\beta+\Theta(\hat{\beta})^{\frac{1}{2}})^3\end{aligned} \tag{3.58}$$

Under the incentive-compatible constraint (it is the publisher's best interest to announce the true market state), we have that $\hat{\beta}=\beta$ and $e(\hat{\beta}=\beta,\beta)=e^*(\beta)=\Theta(\beta)$. Thus, the marginal rents w.r.t the true market state is

$$\begin{aligned}\frac{d}{d\beta}u_p(\beta) &= 4\Theta(\hat{\beta})^{\frac{3}{2}}\\ &= 4e^{*\frac{3}{2}}(\beta)\end{aligned} \tag{3.59}$$

Consider the economic intuition of the publisher's marginal rents. The publisher's marginal rents w.r.t the true market state is $R_1(\beta, e(\hat{\beta},\beta)) + R_2(\beta, e)e_2(\hat{\beta},\beta) - R_1(\beta,0)$. The term $R_1(\beta, e(\hat{\beta},\beta))$ implies the additional return from the spot market when the market state improves. The term $R_2(\beta, e(\hat{\beta},\beta))e_2(\hat{\beta},\beta)$ implies the additional return thanks to the fact that given the announced market situation $\hat{\beta}$, the better the true market situation β, the less effort she has to exert to the traditional contractor, and thus the more expected revenue from the spot market. And the term $R_1(\beta,0)$ implies the hypothetical additional revenue from the spot market as the true market state improves, if the publisher were to exert no effort to the traditional contractor. Consequently, the publisher's marginal rent is the aggregated additional return from the spot market when taking into consideration the opportunity cost (the revenue from the best alternative—devoting all efforts into the spot market). In other words, as the realized market state gets better, the marginal rent for the publisher is just enough to cover the aggregated marginal revenue from the spot market when the true market state improves but the announced market state unchanged. Such design of the marginal rents prevents the publisher from preferring to announce an untruthful market state (or specifically under-report the market state) in order to decrease the effort level she has to exert since the required impression quality is lower if under-report.

According to the publisher's payoff function, the rents for the publisher when the true market state is at its worst scenario, $\beta = \underline{\beta}$, is zero. With the marginal rents is positive, the rents for publisher is increasing with the realized market state.

Concluding the relationship among the induced effort level, the rents for the publisher, and the realized market state, we can state that in the isolated market, both the induced effort level and the publisher's rents are monotonically increasing in the true market state. What's more, when the market state is in its worst scenario, both the induced effort level and the rents for the publisher are zero. When the market state is in its best scenario, the induced effort level is the same as in the socially efficient scenario and the rents for the publisher is positive, and at its highest level.

As we discussed above, that the traditional contractor has to pay the publisher rents in order to have the publisher to announce the true market state, when designing the contract, the traditional contractor must balance between the return from the publisher's effort and the rents he has to pay to the publishers. Thus, at the optimum, it is best for the traditional contractor to induce zero effort from the publisher and pay no rents when the market is in its worst scenario, and induce the socially optimal level of effort and pay the highest level of rents when the market state is in its best scenario.

Expected Payoff To The Traditional Advertiser

Let's consider the expected payoff to the traditional advertiser. In this contracting mechanism, the expected payoff is just the expected return from the impression, $q(\beta, e)$, minus the expected monetary transfer he makes to the publisher, $t(\hat{\beta})$. Under the condition that the publisher announces the true market state, then the expected payoff to the traditional advertiser can be stated as

$$
\begin{aligned}
U_a &= \int_{\underline{\beta}}^{\bar{\beta}} \Big\{x + e^{*\frac{1}{2}}(x) - \int_{\underline{\beta}}^{x} 4e^{*\frac{3}{2}}(t)dt - e^{*}(x)^2\Big\} f(x)dx \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \Big\{x + e^{*\frac{1}{2}}(x) - \frac{8}{5}e^{*\frac{5}{2}}(x) - e^{*2}(x)\Big\} f(x)dx
\end{aligned}
\tag{3.60}
$$

We leave the analysis of the expected payoff of the traditional advertiser with the contract not utilizing spot market pricing information for the moment. We will compare it with the expected payoff with the contract utilizing spot market pricing information later (in section 3.5.4).

3.5.3 Correlated Market

In this section, we will illustrate how a contract utilizing spot market price information yields higher expected payoff for the traditional advertiser and induces a more efficient effort level by the publisher. The impression quality function and the spot market revenue function are the same as in the previous isolated market. What's more, with the true market state, β, and the effort level by the publisher, e, the spot market aggregated price, α, following a Beta distribution on the support of $[\underline{\alpha}, \bar{\alpha}]$ with the mean equal to $\beta - e^2$. Intuitively speaking, when the publisher exerts no effort to the traditional contractor, the spot market price is distributed around the mean of the market state β. When the publisher exerts positive effort e to the traditional contractor, the mean of the distribution for the spot market shifts to the left by the amount of e^2.

In Figure 3.2, we graphically illustrate the market state distribution (β distribution), which is a Beta distribution with both shape parameter a and b equal to 3 on the support of $[0, 1]$. In Figure 3.3, we show the spot market price distribution (α distribution) when the realized market situation equal to different values. In 3.3a, it shows the spot market price distribution when the realized market situation equal to 0.25, $\beta = 0.25$, and the publisher exerts no effort $e = 0$. From the graph, we can tell that when the realized market situation is 0.25, the spot market price distribution is skewed right (positively skewed), with the mean of the distribution equal to 0.25. By assumption, the spot market price follows a Beta distribution on the support of $[-0.16, 1]$[11], but with the mean equal to the realized market situation minus the square of the publisher's effort, $\beta - e^2$. In this case, as $e = 0$, so the mean of the spot market price distribution just equal to β. If the publisher exerts positive effort, the α distribution will be further skewed right by the amount of e^2. In 3.3b and 3.3c, we illustrate the two cases when the realized market situation $\beta = 0.5$ and 0.75, while $e = 0$. We can see that when the realized market situation is just equal to the mean of β distribution (0.5), the α distribution coincides with the β distribution. When the realized market situation is $\beta = 0.75$, the α distribution is skewed left (negatively skewed). And similarly, based on the distribution in the illustrated figures, if the publisher exerts positive effort, the α distribution will be further skewed right.

The contracting mechanism works as follows. Besides the guaranteed payment dependent on the publisher's announced market situation, $t(\hat{\beta})$, the traditional advertiser provides a lottery to the publisher, where the probability of getting the reward is dependent on the realized spot market price α. When the realized spot market price is equal or smaller than a given threshold α^*, then the publisher will get a reward $\bar{m}(\hat{\beta})$, and no reward if the realized spot market aggregated price is greater than the threshold. The magnitude is designed to be dependent on the announced market situation as well. And of course, dependent on the publisher's announcement of market state, there is associated impression quality requirement she has to satisfy, $\hat{q}(\hat{\beta})$.

According to the contracting mechanism for the correlated market developed in the section 3.4, when the lottery threshold is set to α^*, the reward magnitude should be set as

$$\begin{aligned}\bar{m}(\hat{\beta}) &= -\frac{R_2(\hat{\beta}, \Theta(\hat{\beta}))e_2(\hat{\beta}, \hat{\beta}) - \phi(\hat{\beta})}{\frac{d}{d\hat{\beta}}(F_c(\alpha^*|\hat{\beta}, \Theta(\hat{\beta}))) + \frac{d}{de}(F_c(\alpha^*|\hat{\beta}, \Theta(\hat{\beta}))e_2(\hat{\beta}, \hat{\beta}))} \\ &= -\frac{1.0119}{2.0119\frac{d}{dx}(F_c(\alpha^*|\hat{\beta} - 0.16))}\end{aligned} \tag{3.61}$$

[11]Please note that the negative lower bound of the spot market price (-0.16) doesn't mean that the spot market price is negative. Instead, the negative lower bound means that when the realized market state is in its worst case, $\beta = \underline{\beta}$, and when the publisher exerts positive effort to the traditional contractor, then the spot market price could be even lower than the worst market state, $\alpha < \underline{\beta}$.

[12]

The lottery works in the following way. When the realized spot market price is equal or smaller than the given threshold α^*, then the traditional advertiser transfers the reward $\bar{m}(\hat{\beta})$ as a function the announced market state $\hat{\beta}$ to the publisher. When the realized spot market aggregated price is greater than the threshold α^*, then there is no reward for the publisher.

Based on the announced market situation, the guaranteed payment is defined as

$$t(\hat{\beta}) = \int_{\underline{\beta}}^{\hat{\beta}} \phi(\tilde{\beta})d\tilde{\beta} - \bar{m}(\hat{\beta})F_c(\alpha^*|\hat{\beta}, E^*(\hat{\beta})) - R(\hat{\beta}, E^*(\hat{\beta})) + R(\hat{\beta}, 0) \tag{3.62}$$

Given $\phi(\hat{\beta}) = 0$ as discussed in Appendix C.14, the guaranteed payment as a function of the announced market situation $\hat{\beta}$ is [13]

$$\begin{aligned} t(\hat{\beta}) &= -\bar{m}(\hat{\beta})F_c(\alpha^*|\hat{\beta}, E^*(\hat{\beta})) - R(\hat{\beta}, E^*(\hat{\beta})) + R(\hat{\beta}, 0) \\ &= \frac{1.0119}{2.0119\frac{d}{dx}(F_c(\alpha^*|\hat{\beta}, 0.4))}F_c(\alpha^*|\hat{\beta}, 0.4) + 0.16 \end{aligned} \tag{3.63}$$

where $\Theta(\hat{\beta}) = 0.4$.

In Figure 3.4, we explicitly illustrate the relationship between the reward magnitude, the guaranteed payment, and the publisher's announced market situation, $\hat{\beta}$. Let's consider the specific case when the spot market price distribution follows a Beta distribution. The mean of the spot market price distribution is defined as $\beta - e^2$, and the interval of the distribution is $[-0.16, 1]$. We take the spot market price threshold, $\alpha^* = 0.5$, as a specific example. We show the associated reward magnitude, the guaranteed payment and together with the winning probability if the publisher announces the true market situation, $\hat{\beta} = \beta$, in the Figure 3.4, where the independent variable is both the announced market state, $\hat{\beta}$, and the true market state, β, as $\hat{\beta} = \beta$.

[12] For a detailed derivation of the reward magnitude, please refer to Appendix C.14.

[13] For a detailed derivation of the guaranteed payment function, please refer to Appendix C.13.

The dashed curve represents the reward magnitude as the announced market state varies. The dash-dotted curve represents how the guaranteed payment varies as the announced market state varies. Then the solid curve show how the winning probability changes as both the announced and true market state vary simultaneously.

The relationship among the three curves is as the following. Under the condition that the publisher announces the true market state, then the expected earnings from this contract must equal to the cost of exerting the equilibrium effort. The expected earnings are defined such that the winning probability times the reward magnitude plus the guaranteed payment. We can tell from the graph that to the left end of the horizontal axis, as the winning probability is close to 1, the reward magnitude and the guaranteed payment is close to the opposite of each other. When to the right end of the horizontal axis, as the winning probability becomes smaller (closer to 0), the reward magnitude is larger than the guaranteed payment with respect to magnitude.

The effort level the publisher must exert when she announced market state $\hat{\beta}$, and when the true market state is β, is defined as follows. When the publisher announces its market state, $\hat{\beta}$, there is an associated impression quality requirement that the publisher must satisfy, which is defined as

$$\begin{aligned} \hat{q}(\hat{\beta}) &= \hat{\beta} + \Theta(\hat{\beta})^{\frac{1}{2}} \\ &= \hat{\beta} + 0.63 \end{aligned} \tag{3.64}$$

Thus, the publisher must satisfy the following condition, in order to meet the above requirement.

$$\hat{\beta} + 0.63 = \beta + e^{\frac{1}{2}} \tag{3.65}$$

Rearranging the above equation, we have the effort level that the publisher has to exert as a function of both the announced market state $\hat{\beta}$, and the true market state β.

$$e = (\hat{\beta} - \beta + 0.63)^2 \tag{3.66}$$

As for the publisher, her expected payoff is the guaranteed payment plus the expected reward from the lottery given her announced market state, plus the expected spot market revenue, minus the outside opportunity (the spot market revenue if the publisher were to exert no effort to the traditional contract), which can be stated as

$$\begin{aligned}\max_{\hat{\beta}} U_p &= t(\hat{\beta}) + \bar{m}(\hat{\beta})F_c(\alpha^*|\beta, e) + R(\beta, e) - R(\beta, 0) \\ &= \frac{1.0119}{2.0119\frac{d}{dx}(F_c(\alpha^*|\hat{\beta}, 0.4))}(F_c(\alpha^*|\hat{\beta}, 0.4) - F_c(\alpha^*|\beta, e)) + 0.16 - e^2 \\ &= \frac{1.0119}{2.0119\frac{d}{dx}(F_c(\alpha^*|\hat{\beta} - 0.4^2))}[F_c(\alpha^*|\hat{\beta} - 0.4^2) - F_c(\alpha^*|\beta - (\hat{\beta} - \beta + 0.63)^4] \\ &+ 0.16 - (\hat{\beta} - \beta + 0.63)^4\end{aligned} \tag{3.67}$$

Maximizing the publisher's objective function w.r.t announced market situation $\hat{\beta}$, we have the F.O.C

$$\begin{aligned}&\frac{d}{d\hat{\beta}}[\frac{1.0119}{2.0119\frac{d}{dx}(F_c(\alpha^*|\hat{\beta} - 0.4^2))}][F_c(\alpha^*|\hat{\beta} - 0.4^2) - F_c(\alpha^*|\beta - (\hat{\beta} - \beta + 0.63)^4] \\ &+ \frac{1.0119}{2.0119\frac{d}{dx}(F_c(\alpha^*|\hat{\beta} - 0.4^2))}[\frac{d}{dx}F_c(\alpha^*|\hat{\beta} - 0.4^2) - \frac{d}{dx}F_c(\alpha^*|\beta - (\hat{\beta} - \beta + 0.63)^4)(-4(\hat{\beta} - \beta + 0.63)^3] \\ &= 4(\hat{\beta} - \beta + 0.63)^3\end{aligned} \tag{3.68}$$

where $\frac{d}{dx}F_c(\alpha^*|x)$ denote the derivative of α's cumulative distribution function w.r.t the mean.

We can verify that $\hat{\beta} = \beta$ makes the above F.O.C to hold. Intuitively, the lottery, the guaranteed payment, and the associated impression quality requirement are designed such that it is the publisher's best interest to announce the true realized market state and meet the associated quality requirement. In Figure 3.5, we show that when the true market state, $\beta = 0.5$, how the publisher's expected earnings, cost of effort, and the expected payment varies as the publisher announces various market state ($\hat{\beta}$ equal or not equal to β).

The dotted curve illustrates the effort level the publisher must exert when she announces $\hat{\beta}$, and when the true market state is β. The dashed curve represents the cost of exerting the effort if the publisher reports $\hat{\beta}$. The dash-dotted curve represents the expected earnings from the contract when the publisher reports various $\hat{\beta}$. The solid curve illustrates the expected payoff (expected earnings minus cost) as the publisher reports different market state $\hat{\beta}$. In the Figure 3.5, we can tell that when the true market state is $\beta = 0.5$, announcing $\hat{\beta} = 0.5$ makes the expected payoff (expected rents) for the publisher the highest, around 0, illustrated in the yellow circle in the graph. The associated effort level when announcing the true market state is $e = 0.4$, depicted by the blue circle in the graph.

Let's confirm the results mathematically. Given the impression quality requirement, the effort level at equilibrium when the publisher announcing the true market situation is

$$\begin{aligned} e^* &= (\hat{\beta} - \beta + 0.63)^2 \\ &= 0.4 \end{aligned} \tag{3.69}$$

which is consistent with the Figure 3.5.

Rearrange the above equation, we have the relationship

$$\frac{1}{2}e^{*-\frac{1}{2}} - 2e^* = 0 \tag{3.70}$$

We can tell that the effort induced in this contracting mechanism is identical to the socially efficiently level of effort (3.51). Recall the publisher's expected payoff function, the rents for the publisher at equilibrium is

$$\begin{aligned} u_p &= \frac{F_c(\alpha^*|\beta, 0.4) - F_c(\alpha^*|\beta, 0.4)}{2.26\frac{d}{dx}F_c(\alpha^*|\beta, 0.4)} + 0.16 - 0.4^2 \\ &= 0 \end{aligned} \tag{3.71}$$

So the publisher's rents are equal to zero at equilibrium, which is consistent with the Figure 3.5 as well.

Now, let's discuss how the truth-telling mechanism works. Recall the publisher's marginal rents under the truth-telling condition w.r.t the true market state.

$$\begin{aligned}\frac{d}{d\beta}U_p(\hat{\beta}=\beta,\beta) &= \bar{m}(\beta)[\frac{d}{d\beta}(F_c(\alpha^*|\beta,e(\beta,\beta)) + \frac{d}{de}(F_c(\alpha^*|\beta,e(\beta,\beta))e_2(\beta,\beta))] \\ &\quad + R_1(\beta,e(\beta,\beta)) + R_2(\beta,e(\beta,\beta))e_2(\beta,\beta) - R_1(\beta,0) \\ &= \bar{m}(\beta)[\frac{d}{d\beta}(F_c(\alpha^*|\beta,e(\beta,\beta)) + \frac{d}{de}(F_c(\alpha^*|\beta,e(\beta,\beta))e_2(\beta,\beta))] + R_2(\beta,e(\beta,\beta))e_2(\beta,\beta)\end{aligned} \tag{3.72}$$

As discussed in the isolated market example, the term $R_1(\beta,e(\beta,\beta))+R_2(\beta,e(\beta,\beta))e_2(\beta,\beta)-R_1(\beta,0)$ is the aggregated marginal return from the spot market as the true market state improves ($\beta' = \beta + \Delta\beta$) while the announced market state is still at its original level ($\hat{\beta}=\beta$). What's more, the term $\bar{m}(\beta)[\frac{d}{d\beta}(F_c(\alpha^*|\beta,e(\beta,\beta)) + \frac{d}{de}(F_c(\alpha^*|\beta,e(\beta,\beta))e_2(\beta,\beta))]$ is the marginal return (negative) from the lottery as the true market state improves, holding the announced market situation unchanged. The logic is as follows. As the true market state improves while the announced market state is unchanged, then, while satisfy the original impression quality requirement, the effort level the publisher has to exert is smaller than before. As the effort level, the publisher exerts to the traditional contractor, is less, the better the average impression quality allocated to the spot market. Thus, the spot market impression price level is higher as it is reflecting the true impression quality. Then, the probability the realized spot market aggregated price α to be less the given threshold α^* is smaller. So the probability the publisher can get the reward, $\bar{m}$, is smaller than before. Thus, the marginal return from the lottery as the market situation improves is negative. Consequently, it is the traditional contractor's best interest to design the magnitude of the reward $\bar{m}(\cdot)$ such that the marginal loss from the lottery combined with marginal return from the spot market as the true market state improves to be zero (or approaching zero). Such a method can arbitrarily reduce the marginal rents, which the traditional contractor has to pay to the publisher to prevent her from under-reporting the market state, to the fullest level. Consequently, the rents can be arbitrarily reduced to zero as well.

As the rents, the traditional contractor has to pay, is zero, the traditional contractor's objective function becomes the return from the impression allocated to the traditional contract by the publisher, minus the combined payments to the publisher (guaranteed transfer and the expected reward), which are just enough to cover the publisher's disutility of exerting the effort since the expected rents are zero. Thus, the traditional contractor's objective becomes identical to the social planner's objective. So the induced effort level by this contract is identical to the socially efficient level as well. Thus, we confirm that by utilizing the pricing information from the correlated spot market when designing a contract, the traditional contractor can induce the socially efficient level of effort by the publisher. That is, regardless of the realized market state, the publisher exerts constant relative effort to the traditional contract. Moreover, the traditional contractor can arbitrarily reduce the expected rents he has to pay to the publisher to zero as the information asymmetry has been eliminated by the contract, where part of the payment is contingent on the correlated spot market price information.

Expected Payoff To The Traditional Advertiser

Now consider the expected payoff to the traditional advertiser. In this contracting mechanism which utilizes the price information of the spot market, the expected payoff to the traditional advertiser is the monetary return from the impression (normalize to be quality of the impression), minus the guaranteed payment based the publisher's announced market situation, $\hat{\beta}$, and minus the expected reward (the magnitude of the reward times the probability that the publisher wins) the advertiser pays to the publisher. Thus, under the equilibrium condition that the publisher always announces the true market state $\hat{\beta} = \beta$, we can state the traditional advertiser's expected payoff as

$$
\begin{aligned}
U_a &= \int_{\underline{\beta}}^{\bar{\beta}} \left\{q(x,e^*) - t(x) - \bar{m}(\beta)F_c(\alpha^*|x,e^*)\right\} f(x)dx \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \left\{q(x,e^*) - [-\bar{m}(x)F_c(\alpha^*|x,e^*) - R(x,e^*) + R(x,0) + \bar{m}(x)F_c(\alpha^*|x,e^*)]\right\} f(x)dx \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \left\{x + e^{*\frac{1}{2}} + R(x,e^*) - R(x,0)\right\} f(x)dx \\
&= \int_{\underline{\beta}}^{\bar{\beta}} (x + e^{*\frac{1}{2}} - e^{*2}) f(x)dx \\
&= 0.47 + mean(\beta) \\
&= 0.47 + 0.5
\end{aligned}
\tag{3.73}
$$

since $e^* = 0.4$.

Intuitively, the traditional advertiser's expected payoff is the mean market state plus the maximized social welfare w.r.t the effort level by the publisher ($e^{*\frac{1}{2}} - e^{*2} = 0.47$). We will discuss the comparison between the expected payoffs in the two contracting mechanisms to the traditional advertiser in the next section.

3.5.4 Comparison between the Isolated Market and Correlated Market

Let's first identify the effort level at equilibrium under the two contracting mechanisms. At equilibrium, the effort level in the isolated market is defined by

$$
\frac{1}{2}e_i^{*-1} - 2e_i^{*\frac{1}{2}} = 6\frac{1-F(\beta)}{f(\beta)} \tag{3.74}
$$

The induced effort level in the correlated market is defined by

$$
\frac{1}{2}e_c^{*-\frac{1}{2}} - 2e_c^* = 0 \tag{3.75}
$$

or equivalently

$$\frac{1}{2}e_c^{*-1} - 2e_c^{*\frac{1}{2}} = 0 \tag{3.76}$$

It is obvious to see that the left-hand side terms in both the equations, defining the equilibrium under the two contracting mechanisms, are the same. The right-hand side in the market equilibrium of the isolated case is positive, while the right-hand side in the correlated market equilibrium is zero. Given that the term $\frac{1}{2}e^{*-1} - 2e^{*\frac{1}{2}}$ is monotonically decreasing in the effort level, we can conclude that the induced effort level in the isolated market case, e_i^*, is smaller than the induced effort level in the correlated market case, e_c^*. So we have

$$e_i^* < e_c^* \tag{3.77}$$

As discussed before, in the socially efficient scenario, the effort level also follows the following condition at equilibrium.

$$\frac{1}{2}e^{**-1} - 2e^{**\frac{1}{2}} = 0 \tag{3.78}$$

We can tell that the equilibrium condition for the social planner case is the same as in the correlated market case. Summarizing the discussion, we can state that by utilizing the price information from the correlated spot market when designing a contract between the traditional advertiser and the publisher, the traditional advertiser can induce the socially efficient level of effort by the publisher. However, without utilizing price information from the spot market (like in the isolated market case), the induced effort level by the publisher is lower than the induced effort level in the contract utilizing correlated pricing information from the spot market as well as the socially efficient effort level most of the time (except for the case when $\beta = \bar{\beta}$).

Now let's compare the traditional advertiser's expected payoff between the two contracting mechanisms. Recall the expected payoff for the traditional advertiser under the two contracting mechanisms, respectively. The expected payoff for the traditional advertiser using the contract that doesn't utilize the spot market pricing information is defined as

$$U_{ia} = \int_{\underline{\beta}}^{\bar{\beta}} \{x + e_i^{*\frac{1}{2}}(x) - \frac{8}{5} e_i^{*\frac{5}{2}}(x) - e_i^{*2}(x)\} f(x) dx \tag{3.79}$$

The expected payoff for the traditional advertiser using the contract that utilizes the spot market pricing information is defined as

$$U_{ca} = \int_{\underline{\beta}}^{\bar{\beta}} (x + e_c^{*\frac{1}{2}} - e_c^{*2}) f(x) dx \tag{3.80}$$

It is not difficult to find that comparing the integrands of the advertiser's payoff functions under the two contract mechanisms, there is one negative term in the isolated market case, $-\frac{8}{5}e_i^*(x)$, but not in the correlated market case. What's more, as $e_c^* = e^{**} = 0.4$ is the effort level that maximizes the advertiser's payoff when utilizing spot market price information and also the social planner's objective function, and that e_i^* is always smaller than 0.4 (except for the case when $\beta = \bar{\beta}$), the rest of the integrand for the isolated case is always smaller than the integrand for the correlated market case. $x + e_i^{*\frac{1}{2}}(x) - e_i^{*2}(x) < x + e_c^{*\frac{1}{2}} - e_c^{*2}$. Consequently, the expected payoff for the traditional advertiser is always smaller in the isolated market case compared with the correlated market case. We can conclude that regarding the traditional advertiser's expected payoff, it is the traditional advertiser's best interest to choose the contract that utilizes the spot market price information as it can generate the higher expected payoff for advertiser. The traditional advertiser's expected payoff under such a contracting mechanism is identical to the maximized social welfare, while the publisher is left with zero rents. In figure 3.6, we graphically illustrate the payoffs for the traditional advertiser at optimum dependent on different realized market states under the two contracting mechanisms. We can tell that for any realized market state, β, the expected payoff for the traditional under the contract for the correlated markets (utilizing the price information from the correlated spot market) is always higher than under the contract for the isolated market (not utilizing the price information from the correlated spot market).

3.6 Conclusion

In online display advertising market, the publisher has an information advantage over the traditional contractor regarding the overall market state. This creates a moral hazard problem when the publisher is facing the problem of allocating impressions between the spot market and the traditional contractor. We first formalized the moral hazard problem when implementing the traditional contract mechanism that doesn't utilize price information from the spot market. Then, we focused on solving the moral hazard problem through contracting mechanism design utilizing the information from the spot market bidding prices, which is correlated with the impression quality to the traditional contractor. Our analysis indicates that implementing such a contract, which involves a guaranteed payment as a function of the publisher's announcement of the market state plus a lottery whose reward magnitude is also a function of the publisher's announcement of the market state, can induce the socially efficient level of effort by the publisher. At the same time, by designing the reward magnitude of the lottery as a function of the announced market state, the traditional contractor can arbitrarily reduce the expected rents he must pay to the publisher to almost zero. Moreover, simulation showed that there is a significant loss in terms of social efficiency and the traditional contractor's expected payoff when implementing the contract that doesn't utilize price information from the spot market. However, there is a significant gain in terms of social efficiency and the traditional contractor's expected payoff when implementing the contract that does utilize price information from the spot market.

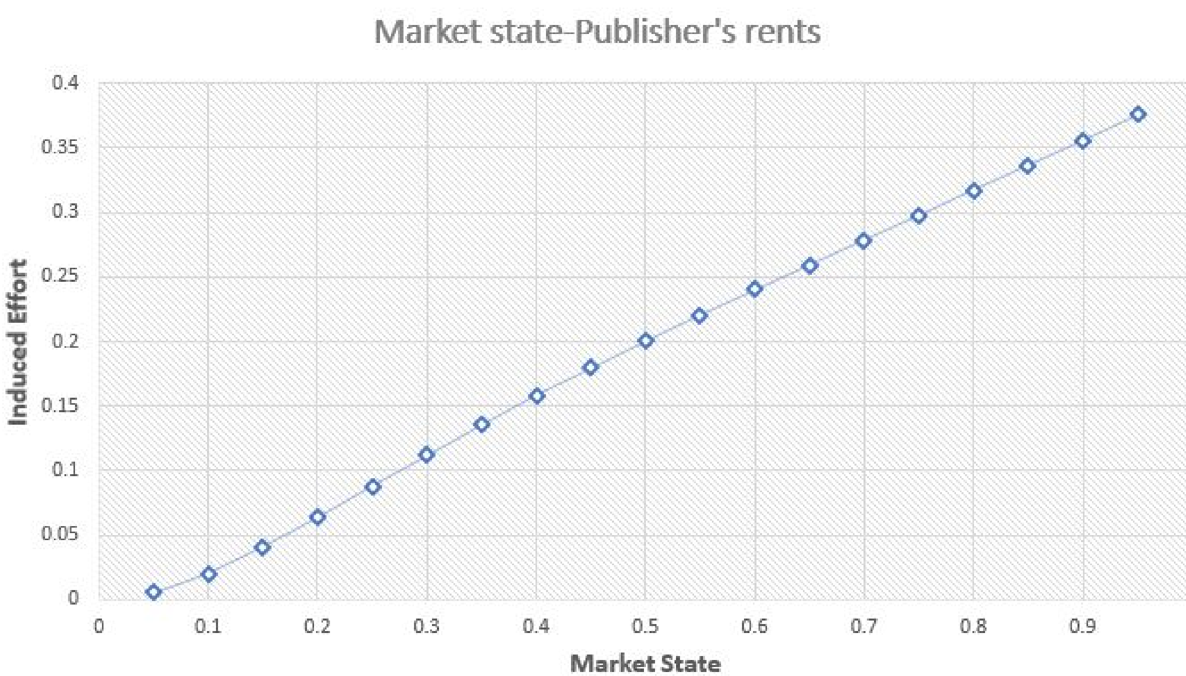

Figure 3.1: Realized market state and induced effort

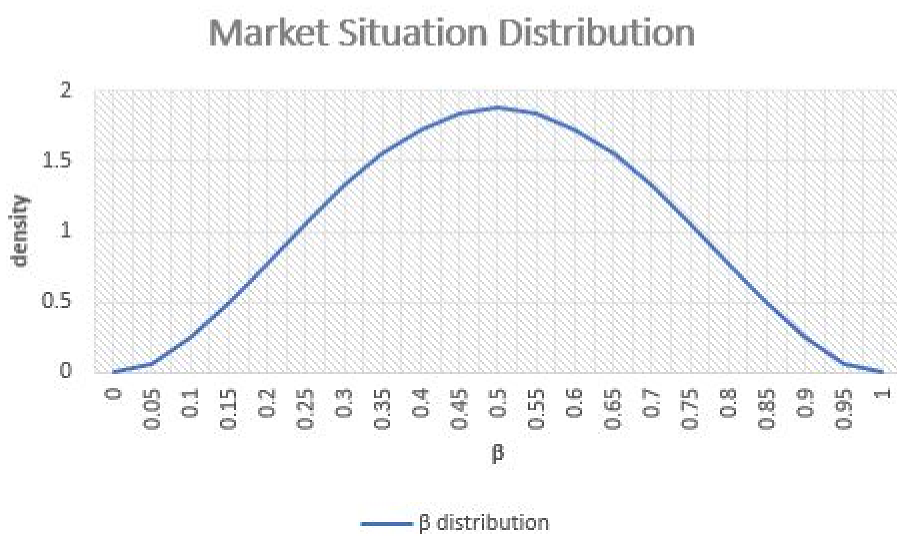

Figure 3.2: Market state distribution

(a) $\beta = 0.25$

(b) $\beta = 0.5$

(c) $\beta = 0.75$

Figure 3.3: Spot market price distributions.

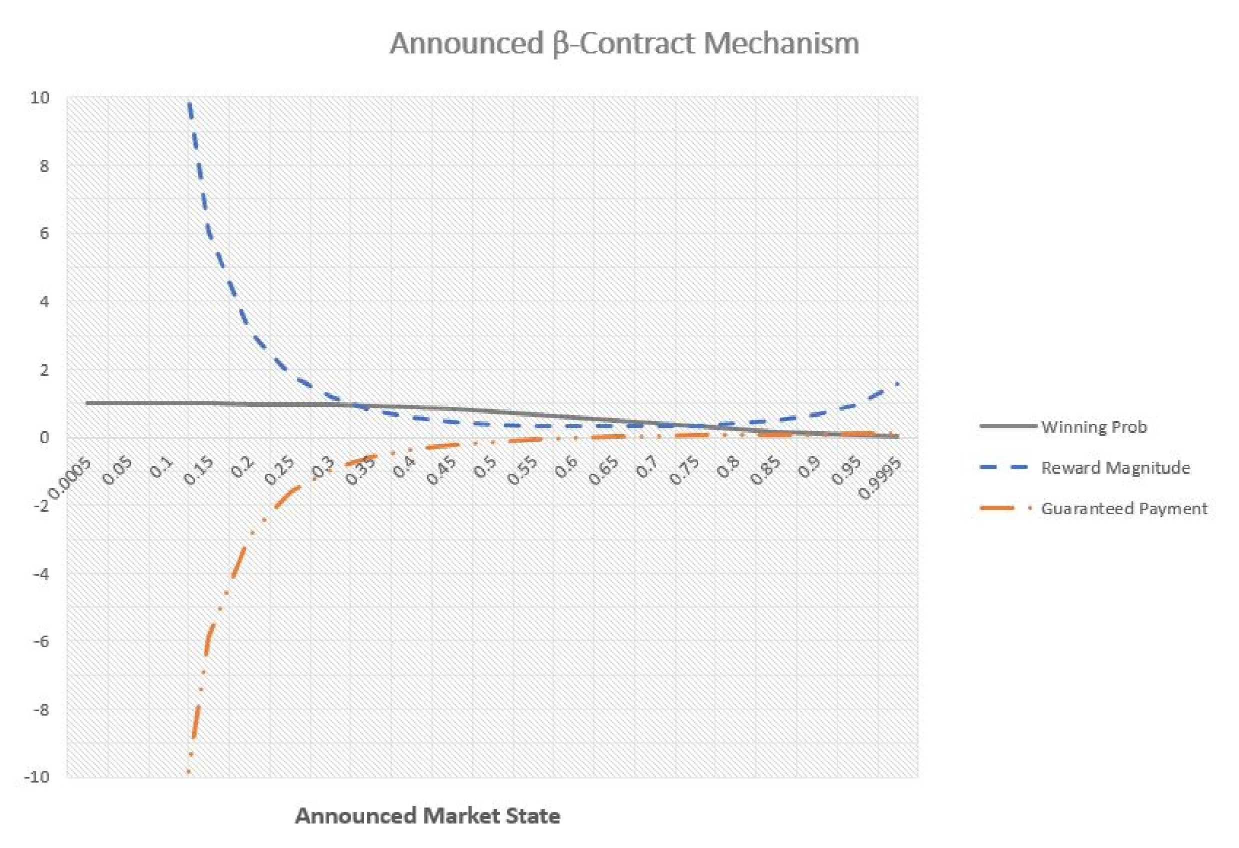

Figure 3.4: Announced market state and contract mechanism

Figure 3.5: Announced market state and publisher expected payoff

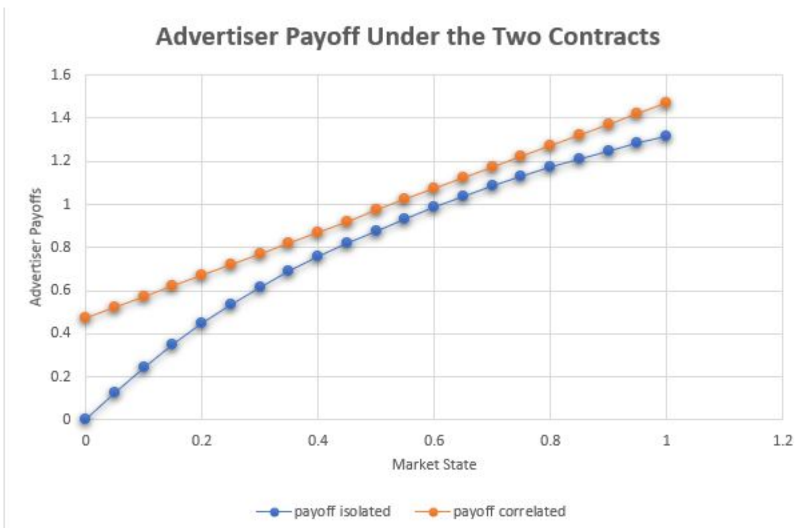

Figure 3.6: Advertiser payoffs depending on market state under the two contracts

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Appendices

A Chapter 1 Appendix

A.1 Proof for symmetric feasible agreements (two player and N player)

Proof. In a two-player setup. Player i and j. Given the definition of the feasible agreement, none of the players has the incentive to deviate from the candidate agreement.

Consider an asymmetric candidate agreement (σ_i^A, σ_j^A) where $\sigma_i^A > \sigma_j^A$.

Given the the weakly increasing monotonicity of the VB best response function $R_i^{VB}(\sigma_{-i}^A)$ (which is shown in Appendix A.2), we have that

$$\sigma_i^{VBBR} \leq \sigma_j^{VBBR} \tag{81}$$

where $\sigma_i^{VBBR} = R_i^{VB}(\sigma_j^j)$ and $\sigma_j^{VBBR} = R_j^{VB}(\sigma_i^A)$.

As $\sigma_i^{VBBR} \leq \sigma_j^{VBBR}$ and $\sigma_i^A > \sigma_j^A$, at least one of the players i and j has incentive to deviate from the candidate agreement. This is contradicting with the definition of feasible agreement. Therefore, no asymmetric candidate agreement can be feasible agreement.

In a N-player setup. Player i, $i \in N$, where $N = 1, ..., n$. In order for a candidate agreement to be feasible agreement, none of the players have incentive to deviate from the candidate agreement.

Consider an asymmetric candidate $(\sigma_1^A, \sigma_2^A, ..., \sigma_n^A)$ such that for a subset of players, $K = 1, ..., k$ and $K \in N$, $\forall g, h \in K, \sigma_g^A \neq \sigma_h^A$. Without loss of generosity, we can rank the subset of unequal candidate agreement contributions as

$$\sigma_1^A < \sigma_2^A < \cdots < \sigma_k^A. \tag{82}$$

Then $\forall l, m \in K$ and $l < m$, we have $\sigma_l^A < \sigma_m^A$. Therefore, $\Delta_{-l} > \Delta_{-m}$, where $\Delta_{-i} = \sigma_1^A * \cdots * \sigma_{i-1}^A * \sigma_{i+1}^A * \cdots * \sigma_k^A$.

According to the weakly increasing VB best response function, $R_i^{VB}(\sigma_{-i})$, we have

$$\sigma_l^{VBBR} \geq \sigma_m^{VBBR} \tag{83}$$

where $\sigma_i^{VBBR} = R_i^{VB}(\Delta_{-i})$.

Since $\sigma_l^A < \sigma_m^A$ and $\sigma_l^{VBBR} \geq \sigma_m^{VBBR}$ are contradicting with the definition of the feasible agreement, no asymmetric candidate agreement can be a feasible agreement.

□

A.2 Proof for weak monotonicity of the Virtual Bargaining best response function

Proof. By maximizing player i's payoff given other players' strategies, we have player i's Nash best response function as the following.

$$\sigma_i^{NBR} = R_i^{NBR}(\sigma_{-i}) = (\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} (\prod_{j\neq i}(\sigma_j))^{-\frac{\alpha}{\alpha-n}} \tag{84}$$

To simplify the equation, set $\Delta_{-i} = \prod_{j\neq i}(\sigma_j)$.

Then, we have

$$\sigma_i^{NBR} = R_i^{NBR}(\Delta_{-i}) = (\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} \Delta_{-i}{}^{-\frac{\alpha}{\alpha-n}} \tag{85}$$

Given that $0 < \alpha < 1$, and n is positive integer, it is obvious that the Nash best response function R_i^{NBR} is monotonically increasing in Δ_{-i}.

Given a candidate agreement $(\sigma_1^A, \cdots, \sigma_n^A)$, we have $\Delta_{-i}^A = \prod_{j\neq i}(\sigma_j^A)$. Consider the case when all other players $-i$ follow through the candidatea agreement $(\sigma_1^A, \cdots, \sigma_n^A)$, then player i's best response function is $R_i^{NBR}(\Delta_{-i}^A)$.

Suppose we have $\Delta_{-i}^{A'} > \Delta_{-i}{}^A$. Given that function R^{NBR} is monotonically increasing, then we have

$$R_i^{NBR}(\Delta_{-i}^{A'}) > R_i^{NBR}(\Delta_{-i}^A) \tag{86}$$

Thus player i's best response is monotonically increasing in the production of all other players' contributions in case when all other players following throught the candidate agreement.

Now set $M = N - i$, denoting all players except i. We consider the case when a subset of all other players, $K \in M$ best respond to the candidate agreement.

Given a candidate agreement $(\sigma_1^A, \cdots, \sigma_n^A)$, if a subset, $K \in M$, of all other players M best respond to the candidate agreement, we have player k's best response function given other players' candidate agreement strategies $\sigma_g^A, g \neq k$ as follows.

$$\begin{aligned} R_k^{NBR}(\sigma_{-k}^A) =& (\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} {\Delta_{-k}^A}^{-\frac{\alpha}{\alpha-n}} \\ & (\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} \prod_{g \neq k} (\sigma_g^A)^{-\frac{\alpha}{\alpha-n}} \quad \forall k \in K \end{aligned} \tag{87}$$

Then, we have the product of all best responding players' contributions as

$$\begin{aligned} T_{k\in K}(\sigma_{-k}^A) = & \prod_{k\in K} R_k^{NBR}(\sigma_{-k}^A) \\ & (\frac{\alpha}{n})^{-\frac{nk}{\alpha-n}} \cdot M^{-\frac{nk}{\alpha-n}} (\prod_{p\in N-K} {\sigma_p^A}^k \cdot \prod_{q\in K} {\sigma_q^A}^{k-1})^{-\frac{\alpha}{\alpha-n}} \\ & (\frac{\alpha}{n})^{-\frac{nk}{\alpha-n}} \cdot M^{-\frac{nk}{\alpha-n}} (\prod_{p\in M-K} \sigma_p^A \cdot \prod_{s\in M} {\sigma_s^A}^{k-1} \cdot {\sigma_i^A}^{k-1})^{-\frac{\alpha}{\alpha-n}} \\ & (\frac{\alpha}{n})^{-\frac{nk}{\alpha-n}} \cdot M^{-\frac{nk}{\alpha-n}} \cdot \prod_{p\in M-K} {\sigma_p^A}^{-\frac{\alpha}{\alpha-n}} \cdot \prod_{s\in M} {\sigma_s^A}^{-\frac{\alpha(k-1)}{\alpha-n}} \cdot {\sigma_i^A}^{-\frac{\alpha(k-1)}{\alpha-n}} \end{aligned} \tag{88}$$

Thus, given that a subset of players, $K \in M$, best repond to the candidate agreement $(\sigma_1^A, \cdots, \sigma_n^A)$, the product of all other players $-i$'s contribution is

$$\Delta_{-i}^{BR_k}(\sigma_{-i}^A) = (\prod_{p\in M-K} \sigma_p^A) T_{k\in K}(\sigma_k^A) \tag{89}$$

Given $0 < \alpha < 1$, and n is integer greater or equal to 2, function T is weakly increasing in contribution of any player other than player i, σ_{-i}^A, so is the function, $\Delta_{-i}^{BR_k}$.

Player i's object function is

$$\max_{\sigma_i} u_i(\sigma_i^A, \Delta_i^{BR_k}) = y_i - \sigma_i^A + M{\sigma_i^A}^{\frac{\alpha}{n}} {\Delta_{-i}^{BR_k}}^{\frac{\alpha}{n}} \tag{90}$$

Given that by maximizing $u_i(\sigma_i^A, \Delta_{-i})$ over σ_i^A, the best response function $R_i^{NBR}(\Delta_{-i})$ is monotonically increasing in Δ_{-i}, and we have proven that function $\Delta_{-i}^{BR_k}(\sigma_{-i}^A)$ is weakly increasing in σ_{-i}^A. We can conclude that function $r_i(\sigma_{-i}^A) = R_i^{NBR}(\Delta_{-i}^{BR_k}(\sigma_{-i}^A))$ is weakly increasing in σ_{-i}^A.

Since player i's best response is weakly increasing in any of all other players $-i$' contributions in candidate agreement, σ_{-i}^A, regardless of how many other players are best responding or following through the candidate agreement, player i's VB best response function is weakly increasing in any of other players $-i$'s contribution, σ_{-i}^A.

□

A.3 Derivation of VB best response function Eq(1.8)

Consider the properties of $w_i(\sigma_i^A, \Delta)$ as a function of σ_i^A. When $\Delta \leq \frac{\alpha}{n}^{-\frac{n-1}{\alpha-1}} M^{-\frac{n-1}{\alpha-1}}$, $w_i(\sigma_i^A, \sigma_{-i}^A)$ as a function of σ_i^A, increases on the interval $[0, (\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}})$, has a kink at $(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}})$, then increases on the interval $[(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}}), (\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} \Delta^{-\frac{\alpha}{\alpha-n}})$, achieves its unique maximum at $(\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} \Delta^{-\frac{\alpha}{\alpha-n}}$, and decreases for the value of σ_i^A exceeding $(\frac{\alpha}{n})^{-\frac{n}{\alpha-n}} M^{-\frac{n}{\alpha-n}} \Delta^{-\frac{\alpha}{\alpha-n}}$.

When $\frac{\alpha}{n}^{-\frac{n-1}{\alpha-1}} M^{-\frac{n-1}{\alpha-1}} < \Delta \leq (-\frac{\alpha n-n^2}{\alpha^2(n-2)+\alpha n})^{\frac{\alpha(n-1)}{\alpha n-n}} \frac{\alpha}{n}^{\frac{(\alpha-n)(n-1)}{\alpha n-n}} M^{-\frac{n-1}{\alpha-1}}$, $w_i(\sigma_i^A, \Delta)$ as a function of σ_i^A, increases on the interval $[0, (\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}})$, achieves its unique maximum at $(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}}$, which is also a kink at $(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}}$, and decreases for the value of σ_i^A exceeding $(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}}$.

When $\sigma_i^A > (-\frac{\alpha n-n^2}{\alpha^2(n-2)+\alpha n})^{\frac{\alpha(n-1)}{\alpha n-n}} \frac{\alpha}{n}^{\frac{(\alpha-n)(n-1)}{\alpha n-n}} M^{-\frac{n-1}{\alpha-1}}$, $w_i(\sigma_i^A, \Delta)$ as a function of σ_i^A, increases on the interval $[0, (-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2})^{\frac{\alpha n-n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \frac{\alpha}{n}^{-\frac{\alpha n(n-1)}{\alpha^2(n-2)+2\alpha n-n^2}} M^{-\frac{\alpha n(n-2)+n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \Delta^{-\frac{\alpha^2(n-2)}{\alpha^2(n-2)+2\alpha n-n^2}})$, achieves its unique maximum at $(-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2})^{\frac{\alpha n-n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \frac{\alpha}{n}^{-\frac{\alpha n(n-1)}{\alpha^2(n-2)+2\alpha n-n^2}} M^{-\frac{\alpha n(n-2)+n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \Delta^{-\frac{\alpha^2(n-2)}{\alpha^2(n-2)+2\alpha n-n^2}}$, then decreases on the interval $[(-\frac{\alpha^2(n-2)+\alpha n}{\alpha n-n^2})^{\frac{\alpha n-n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \frac{\alpha}{n}^{-\frac{\alpha n(n-1)}{\alpha^2(n-2)+2\alpha n-n^2}} M^{-\frac{\alpha n(n-2)+n^2}{\alpha^2(n-2)+2\alpha n-n^2}} \Delta^{-\frac{\alpha^2(n-2)}{\alpha^2(n-2)+2\alpha n-n^2}}$,

$(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}})$, has a kink at $(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}}$, and continue to decrease for the value of σ_i^A exceeding $(\frac{\alpha}{n})^{-\frac{n}{\alpha}} M^{-\frac{n}{\alpha}} \Delta^{-\frac{\alpha(n-1)-n}{\alpha(n-1)}}$.

Note that $w_i(\sigma_i^A, \Delta)$ is single peaked as a function of σ_i^A, in all three cases characterized in the preceding paragraph. The unique peak in each of these cases corresponds to the best-response function in Eq (1.8).

A.4 Proof for $\sigma^{VB} > \sigma^{NE}$ in teamwork game.

Given that

$$\sigma^{NE} = (\frac{\alpha}{n})^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}} \tag{91}$$

and

$$\sigma^{VB} = (-\frac{\alpha^2(n-2)+\alpha n}{\alpha n - n^2})^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}} (\frac{\alpha}{n})^{-\frac{\alpha(n-1)}{(\alpha-1)(\alpha(n-2)+n)}} M^{-\frac{1}{\alpha-1}} \tag{92}$$

,

we define

$$\begin{aligned}
\frac{\sigma^{VB}}{\sigma^{NE}} &= (-\frac{\alpha^2(n-2)+\alpha n}{\alpha n - n^2})^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}} (\frac{\alpha}{n})^{-\frac{\alpha(n-1)}{(\alpha-1)(\alpha(n-2)+n)}} M^{-\frac{1}{\alpha-1}} \boldsymbol{\nabla} \boldsymbol{\cdot} \Big[(\frac{\alpha}{n})^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}} \Big] \\
&= (-\frac{\alpha^2(n-2)+\alpha n}{\alpha n - n^2})^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}} (\frac{\alpha}{n})^{\frac{n-\alpha}{(\alpha-1)(\alpha(n-2)+n)}} \\
&= (-\frac{\alpha n - n^2}{\alpha^2(n-2)+\alpha n})^{\frac{n-\alpha}{(\alpha-1)(\alpha(n-2)+n)}} (\frac{\alpha}{n})^{\frac{n-\alpha}{(\alpha-1)(\alpha(n-2)+n)}} \\
&= (\frac{n-\alpha}{\alpha(n-2)+n})^{\frac{n-\alpha}{(\alpha-1)(\alpha(n-2)+n)}} \\
&= \Big\{ (\frac{n-\alpha}{\alpha(n-2)+n})^{\frac{n-\alpha}{\alpha(n-2)+n}} \Big\}^{\frac{1}{\alpha-1}}
\end{aligned} \tag{93}$$

The term

$$\frac{n-\alpha}{\alpha(n-2)+n} = \frac{1}{\frac{\alpha(n-1)}{n-\alpha}+1} \tag{94}$$

Given that $0 < \alpha < 1$ and $n \geq 2, n \in \mathbb{Z}$, we have

$$0 < \frac{1}{\frac{\alpha(n-1)}{n-\alpha}+1} < 1 \tag{95}$$

Thus, we have

$$0 < \left\{\frac{1}{\frac{\alpha(n-1)}{n-\alpha}+1}\right\}^{\frac{1}{\frac{\alpha(n-1)}{n-\alpha}+1}} < 1 \tag{96}$$

or, equivalently,

$$0 < \left\{(\frac{n-\alpha}{\alpha(n-2)+n})^{\frac{n-\alpha}{\alpha(n-2)+n}}\right\} < 1 \tag{97}$$

And given $\frac{1}{\alpha-1} < -1$, we have

$$\left\{(\frac{n-\alpha}{\alpha(n-2)+n})^{\frac{n-\alpha}{\alpha(n-2)+n}}\right\}^{\frac{1}{\alpha-1}} > 1 \tag{98}$$

Thus, we have showed that

$$\sigma^{VB} > \sigma^{NE} \tag{99}$$

A.5 Proof for Proposition 1.4

Proof. Given that $\sigma^{SO} = \alpha^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$, the socially optimal level of contribution is independent of number of players, n.

Given that $\sigma^{NE} = (\frac{\alpha}{n})^{-\frac{1}{\alpha-1}} M^{-\frac{1}{\alpha-1}}$, the Nash equilibrium level of contribution is decreasing in the number of players, n.

The VB equilibrium contribution can be stated as follows.

$$
\begin{aligned}
\sigma^{VB} &= (-\frac{\alpha^2(n-2)+\alpha n}{\alpha n - n^2})^{\frac{\alpha-n}{(\alpha-1)(\alpha(n-2)+n)}}(\frac{\alpha}{n})^{-\frac{\alpha(n-1)}{(\alpha-1)(\alpha(n-2)+n)}}M^{-\frac{1}{\alpha-1}} \\
&= (\frac{\alpha}{n})^{\frac{(1+\alpha)n-2\alpha}{(1-\alpha)[\alpha(n-2)+n]}}(\frac{\alpha(n-2)+n}{n-\alpha})^{\frac{n-\alpha}{(1-\alpha)[\alpha(n-2)+n]}}M^{-\frac{1}{\alpha-1}} \\
&= (\frac{\alpha}{n})^{\frac{1}{1-\alpha}}(\frac{\alpha(n-2)+n}{n-\alpha})^{\frac{n-\alpha}{(1-\alpha)[\alpha(n-2)+n]}}M^{-\frac{1}{\alpha-1}}
\end{aligned}
$$

Since the exponential term $\frac{(1+\alpha)n-2\alpha}{(1-\alpha)[\alpha(n-2)+n]}$ can be simplified as follows.

$$
\begin{aligned}
\frac{(1+\alpha)n-2\alpha}{(1-\alpha)[\alpha(n-2)+n]} &= \frac{(1+\alpha)n-2\alpha}{(\alpha-\alpha^2)n-2(\alpha-\alpha^2)+(1-\alpha)n} \\
&= \frac{(1+\alpha)n-2\alpha}{(1-\alpha)(1+\alpha)n-2\alpha(1-\alpha)} \\
&= \frac{(1+\alpha)n-2\alpha}{(1-\alpha)[(1+\alpha)n-2\alpha]} \\
&= \frac{1}{1-\alpha}
\end{aligned}
$$

Then, the VB equilibrium can be restated as

$$
\begin{aligned}
\sigma^{VB} &= (\frac{\alpha}{n})^{\frac{1}{1-\alpha}}(A^{\frac{1}{A}})^{\frac{1}{1-\alpha}}M^{\frac{1}{1-\alpha}} \\
&= (\frac{\alpha}{n}A^{\frac{1}{A}})^{\frac{1}{1-\alpha}}M^{\frac{1}{1-\alpha}}
\end{aligned}
$$

where $A = \frac{\alpha(n-2)+n}{n-\alpha}$.

In order to determine $\frac{d}{dn}\sigma^{VB}$, it is equivalent to determine $\frac{d}{dn}(\frac{\alpha}{n}A^{\frac{1}{A}})$. Let

$$
B = \frac{\alpha}{n}A^{\frac{1}{A}} = \frac{\alpha}{n}(\frac{\alpha(n-2)+n}{n-\alpha})^{\frac{n-\alpha}{\alpha(n-2)+n}}
$$

Differentiate B w.r.t n, we have

$$\frac{d}{dn}B = -\frac{\{\alpha(1-\alpha)n\ln\frac{\alpha(n-2)+n}{n-\alpha} + (\alpha+1)^2n^2 - (3\alpha^2+5\alpha)n + 4\alpha^2\}\alpha(\frac{\alpha(n-2)+n}{n-\alpha})^{\frac{n-\alpha}{\alpha(n-2)+n}}}{n^2((\alpha+1)n-2\alpha)^2}$$

Now, let consider the term $A = \frac{\alpha(n-2)+n}{n-\alpha}$.

$$\begin{aligned} A &= \frac{(1+\alpha)n-2\alpha}{n-\alpha} \\ &= \alpha+1-\frac{\alpha(1-\alpha)}{n-\alpha} \end{aligned}$$

Given $0<\alpha<1$, it is evident that A is increasing in n. $A=\frac{2}{2-\alpha}>1$ when $n=2$. $A=\alpha+1<2$ when $n\to\infty$. Thus, we have $1<A<2$. Then, we have $\ln\frac{\alpha(n-2)+n}{n-\alpha}=\ln A>0$. What's more, it is evident that the denominator of $\frac{d}{dn}B$ is positive, and that the terms $\alpha(1-\alpha)n\ln\frac{\alpha(n-2)+n}{n-\alpha}$ and $\alpha(\frac{\alpha(n-2)+n}{n-\alpha})^{\frac{n-\alpha}{\alpha(n-2)+n}}$ in the numerator are positive. Consequently, as long as the term $(\alpha+1)^2n^2-(3\alpha^2+5\alpha)n+4\alpha^2$ is positve, we have $\frac{d}{dn}B<0$.

Let's consider $C=(\alpha+1)^2n^2-(3\alpha^2+5\alpha)n+4\alpha^2$. C is first decreasing in n, reaches its lowest value at $\frac{3\alpha^2+5\alpha}{2(\alpha+1)^2}$, and then increasing in n. Given $0<\alpha<1$, we have $0<\frac{3\alpha^2+5\alpha}{2(\alpha+1)^2}<2$. Thus, we have that C is monotonically increasing in n when $n\geq 2$. What's more, we have that when $n=2$

$$C = 2\alpha^2-2\alpha+4>0$$

Thus, we have that $C>0$ for $n\geq 2$. Thus, we have proved that $\frac{d}{dn}B<0$.

Now, let's consider the total outputs under the three cases (Nash equilibrium, VB equilibrium and Socially optimal level). Given the teamwork production technology, $M(\sigma^{NE/VB/SO\frac{\alpha}{n}})^n = M\sigma^{NE/VB/SO\alpha}$, it is evident that the trend of the total output of the teamwork is consistent with the equilibrium individual contribution level. Thus, the socially optimal total output is independent of the number of players, while the Nash and VB equilibrium total outputs are decreasing in the number of players.

□

B Chapter 2 appendix

B.1 Virtual Bargaining Experiment Instructions

*Instruction for treatment 1

Introduction

Thank you for participating in today’s study. Please follow the instructions carefully. At any time, please feel free to raise your hand if you have a question.

You have been randomly assigned an ID number for this session. You will make decisions using a computer. You will never be asked to reveal your identity to anyone. Your name will never be associated with any of your decisions. In order to keep your decisions private, please do not reveal your choices or otherwise communicate with any other participant. Importantly, please refrain from verbally reacting to events that occur.

Today’s session has three parts: Experiment 1, Experiment 2, and a short questionnaire. You will have the opportunity to earn money in both experiments based on your decisions. You will be paid your earnings privately, and in cash, at the end of the experiment session. **We will proceed through the written materials together. Please do not enter any decisions on the computer until instructed to do so.**

Instructions for Experiment 1

Please refer to your computer screen while we read the instructions.

We would like you to make a decision for each of 10 scenarios. Each scenario involves a choice between playing a lottery that pays either $4 or $0 according to specified chances (Option A) or receiving $2 for sure (Option B).

You will notice that the only differences across scenarios are the chances of receiving the high or low prize for the lottery. At the end of the today’s session, ONE of the 10 scenarios will be selected at random and you will be paid according to your decision for this selected scenario ONLY. Each scenario has an equal chance of being selected.

Please consider your choice for each scenario carefully. Since you do not know which scenario will be played out, it is in your best interest to treat each scenario as if it will be the one used to determine your earnings.

Before making decisions, are there any questions?

Please proceed to entering decisions on your computer. Once you are ready to submit your decisions, please click the “Submit” button.

Instructions for Experiment 2

In this experiment, you will be randomly placed into a group of **2 people**. Each group member is given 12 tokens. These tokens can either be kept or contributed to a **team project.** The contributions to the team project (from you and the other group member) yields a payout to both team members. This payout is the same for each member. Each token you do not contribute to the team project belongs only to you. Your total income will equal your income from the team project plus the number of tokens you kept. The exchange rate for this experiment will be 25 tokens to 1 US dollar.

Your Earnings from the Team Project

You and the other group member will earn the same amount of money from the team project, even if you contribute different amounts. The earnings from the team project is determined by the formula provided below, which depends on how much you and the other player contribute:

$$\boldsymbol{Your\ earnings\ from\ the\ team\ project} = \mathbf{6.5} \times (\boldsymbol{your\ contribution} \times \boldsymbol{contribution\ of\ the\ other\ player})^{\mathbf{0.3}}$$

This formula is a little complicated, so let's discuss what is going on. Know that, with this formula:

- Your earnings from the team project increases with the number of tokens *you* contribute.
- Your earnings from the team project increases with the number of tokens the *other player* contributes.
- The *additional* earnings you earn from the team project by contributing *another* token increases when the other player contributes more.

Let's work through an example to see what we mean by the third bullet point. Suppose the other player contributes 1 token and you are deciding whether to contribute 4 or 5 tokens. In the first case, you would earn $6.5 \times (4 \times 1)^{0.3} = 9.85$ tokens. In the second you would earn $6.5 \times (5 \times 1)^{0.3} = 10.53$ tokens. This means that you would earn an additional **0.68** tokens by contributing five tokens instead of four tokens.

Now suppose the other player contributes 9 tokens. Now if you contribute 4 tokens you would earn $6.5 \times (4 \times 9)^{0.3} = 19.05$, and if you contribute 5 tokens you would earn $6.5 \times (5 \times 9)^{0.3} = 20.36$. You earn an additional **1.31** tokens by contributing five tokens instead of four. The additional income you earn from contributing another token to the team project is now much higher in this example (1.31 tokens instead of 0.68 tokens), as the other player is contributing more.

Your Total Earnings

Your total earnings are the sum of your earnings from the team project and the tokens you kept for yourself. This is the amount you earn, in tokens, from your decision in a particular decision period.

$$\boldsymbol{Your\ total\ earnings = your\ earnings\ from\ the\ team\ project + number\ of\ tokens\ kept}$$

The table below provides a calculation of your **total earnings** for every possible scenario based on what you and the other player contribute. As an example, suppose you contribute 3 tokens and the other player contributes 8 tokens. To find your total earnings for this example, find the row that corresponds with your contribution (3) and the column that corresponds with what the other player contributes (8). This row and column intersect at the entry 25.86. This is your total earnings in this example.

Applying the formula from before, your earnings from the team project are $6.5 \times (3 \times 8)^{0.3} = 16.86$ tokens. Since you contribute 3 tokens, this means you kept 9 tokens for yourself. Therefore, your total earnings are 16.86 (earnings from the team project) plus 9 (number of tokens kept), which equals 25.86.

Please keep in mind that the entries in the table account for both the payout from the team project and the payout from keeping tokens. During the experiment, the computer will display this table. You will also be given an on-screen calculator.

Total Income

	Number of tokens the other player contributes												
Number of tokens you contribute	0	1	2	3	4	5	6	7	8	9	10	11	12
0	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
1	11.00	17.50	19.00	20.04	20.85	21.53	22.13	22.65	23.13	23.57	23.97	24.35	24.70
2	10.00	18.00	19.85	21.13	22.13	22.97	23.70	24.35	24.93	25.47	25.97	26.43	26.86
3	9.00	18.04	20.13	21.57	22.70	23.65	24.47	25.20	25.86	26.47	27.03	27.56	28.05
4	8.00	17.85	20.13	21.70	22.93	23.97	24.86	25.66	26.38	27.05	27.66	28.23	28.76
5	7.00	17.53	19.97	21.65	22.97	24.07	25.03	25.89	26.66	27.36	28.02	28.63	29.20
6	6.00	17.13	19.70	21.47	22.86	24.03	25.05	25.95	26.76	27.51	28.20	28.84	29.45
7	5.00	16.65	19.35	21.20	22.66	23.89	24.95	25.89	26.75	27.53	28.25	28.93	29.56
8	4.00	16.13	18.93	20.86	22.38	23.66	24.76	25.75	26.63	27.45	28.20	28.90	29.56
9	3.00	15.57	18.47	20.47	22.05	23.36	24.51	25.53	26.45	27.29	28.07	28.80	29.48
10	2.00	14.97	17.97	20.03	21.66	23.02	24.20	25.25	26.20	27.07	27.88	28.63	29.33
11	1.00	14.35	17.43	19.56	21.23	22.63	23.84	24.93	25.90	26.80	27.63	28.40	29.12
12	0.00	13.70	16.86	19.05	20.76	22.20	23.45	24.56	25.56	26.48	27.33	28.12	28.87

Practice Questions

To help you understand the procedures, we ask that you answer four practice questions using your computer. But here is the good news. For each question you answer correctly, you will earn 10 tokens.

Please click “Continue” on your computer to begin the practice questions. Please raise your hand if you need assistance when answering these questions.

Starting the Experiment

We are now ready to begin the experiment. We will proceed through a large number of decision periods. You will not know the number of decision periods until the experiment has been completed. Each decision period will be exactly as described in the instructions. Each decision period is separate from the other periods, in the sense that the decision you make in one period will not affect the outcome or earnings of any other period.

Prior to the practice round and each of the paid periods, everyone will be randomly assigned into two-player groups. Because of this randomization, you are very unlikely to be paired with the same person in more than one decision period.

We will first go through one <u>unpaid</u> practice period. After this practice period, you will be paid based on each and every decision you make.

Before we continue, do you have any questions?

Once you are ready to proceed, please click the “Continue” button.

*Instruction for treatment 5

Introduction

Thank you for participating in today's study. Please follow the instructions carefully. At any time, please feel free to raise your hand if you have a question.

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Instructions for Experiment 1

Please refer to your computer screen while we read the instructions.

We would like you to make a decision for each of 10 scenarios. Each scenario involves a choice between playing a lottery that pays either $4 or $0 according to specified chances (Option A) or receiving $2 for sure (Option B).

You will notice that the only differences across scenarios are the chances of receiving the high or low prize for the lottery. At the end of the today's session, ONE of the 10 scenarios will be selected at random and you will be paid according to your decision for this selected scenario ONLY. Each scenario has an equal chance of being selected.

Please consider your choice for each scenario carefully. Since you do not know which scenario will be played out, it is in your best interest to treat each scenario as if it will be the one used to determine your earnings.

Before making decisions, are there any questions?

Please proceed to entering decisions on your computer. Once you are ready to submit your decisions, please click the "Submit" button.

Instructions for Experiment 2

In this experiment you will play the role of a firm. You and another player will compete for sales in a product market. Your task is to determine a selling price for your firm's product. This price can be any value between 0 to 12 tokens, in integer (whole number) amounts. Your earnings will be based on the prices chosen by you and the other player. At the end of the experiment, tokens will be converted at a rate of 120 tokens to 1 US dollar.

Your Earnings from the Market

Your earnings from the market will be determined by your decision of price and the price decision of the other seller. Specifically, your earnings are your price times the quantity sold. The earnings from the duopoly market are determined by the formula provided below, which depends on the prices you and the other firm decide:

$$\boldsymbol{Your\ earnings = your\ price\ decision \times quantity\ sold}$$

and

$$\boldsymbol{Quantity\ sold = \ 24.39 - 1.62 \times your\ price\ decision + 0.5 \times the\ other\ firm's\ price\ decision}$$

This formula is a little complicated, so let's discuss what is going on. Know that, with this formula:

- The quantity sold increases with the price the other seller determines, and consequently your earnings from the duopoly market increases with the price the other seller determines
- The quantity sold decreases with the price you determine
- Your earnings from the duopoly market increases first and then decreases with the price you determine

Let's work through an example to see what we mean by the first bullet point. Suppose the you decide the price level to be 6 tokens and the other firm is deciding the price level to be either 5, 6 or 7 tokens. In the first case, the quantity sold is $24.39 - 1.62 \times \mathbf{6} + \mathbf{0.5} \times \mathbf{5} = \mathbf{17.17}$, and your earnings are $\mathbf{6} \times \mathbf{17.17} = \mathbf{103.02}$ tokens. In the second case, the quantity sold is $24.39 - 1.62 \times \mathbf{6} + \mathbf{0.5} \times \mathbf{6} = \mathbf{17.67}$, and your earnings are $\mathbf{6} \times \mathbf{17.67} = \mathbf{106.02}$ tokens. In the third case, the quantity sold is $24.39 - 1.62 \times \mathbf{6} + \mathbf{0.5} \times \mathbf{7} = \mathbf{18.17}$, and your earnings are $\mathbf{6} \times \mathbf{18.17} = \mathbf{109.02}$ tokens. As the other firm increases price from 5 to 6 to 7 tokens, the quantity sold increases from 17.17 to 17.67 to 18.17, and your earnings increase from 103.02 to 106.02 to 109.02 tokens.

Now let's work through an example to see what we mean by the second and third bullet point. Suppose the other firm decides the price to be 8 tokens and you are deciding the price to be 8, 9 or 10 tokens. In the first case, the quantity sold is $24.39 - 1.62 \times 8 + 0.5 \times 8 = \mathbf{15.43}$, and your earnings are $8 \times 15.43 = \mathbf{123.44}$ tokens. In the second case, the quantity sold is $24.39 - 1.62 \times \mathbf{9} + \mathbf{0.5} \times \mathbf{8} = \mathbf{13.81}$, and your earnings are $9 \times 13.81 = \mathbf{124.29}$ tokens. In the third case, the quantity sold from the market is $24.39 - 1.62 \times \mathbf{10} + \mathbf{0.5} \times \mathbf{8} = \mathbf{12.19}$, and your earnings are $10 \times 12.19 = \mathbf{121.9}$ tokens. As you increase the price from 8 to 9 to 10 tokens, the quantity sold decreases from 15.43 to 13.81 and then to 12.19 and your earnings increase from 123.44 tokens to 124.59 tokens first and then decrease to 121.9 tokens.

The table below provides a calculation of your **earnings** for every possible scenario based on the prices that you and the other firm decide. As an example, suppose you decide the price to be 3 tokens and the other firm decides the price to be 8 tokens. To find your earnings for this example, find the row that corresponds with your price decision (3) and the column that corresponds with the other firm's price decision (8). This row and column intersect at the entry 70.59. There are your earnings in this example.

Applying the formula from before, your quantity sold is $24.39 - 1.62 \times 3 + 0.5 \times 8 = 23.53$. Since your price decision is 3 tokens, then your earnings are $3 \times 23.53 = 70.59$ tokens.

During the experiment, the computer will display this table. You will also be given an on-screen calculator.

Total Income

		Price decision of the other firm												
		0	1	2	3	4	5	6	7	8	9	10	11	12
Your price decision	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	22.80	23.30	23.80	24.30	24.80	25.30	25.80	26.30	26.80	27.30	27.80	28.30	28.80
	2	42.40	43.40	44.40	45.40	46.40	47.40	48.40	49.40	50.40	51.40	52.40	53.40	54.40
	3	58.80	60.30	61.80	63.30	64.80	66.30	67.80	69.30	70.80	72.30	73.80	75.30	76.80
	4	72.00	74.00	76.00	78.00	80.00	82.00	84.00	86.00	88.00	90.00	92.00	94.00	96.00
	5	82.00	84.50	87.00	89.50	92.00	94.50	97.00	99.50	102.00	104.50	107.00	109.50	112.00
	6	88.80	91.80	94.80	97.80	100.80	103.80	106.80	109.80	112.80	115.80	118.80	121.80	124.80
	7	92.40	95.90	99.40	102.90	106.40	109.90	113.40	116.90	120.40	123.90	127.40	130.90	134.40
	8	92.80	96.80	100.80	104.80	108.80	112.80	116.80	120.80	124.80	128.80	132.80	136.80	140.80
	9	90.00	94.50	99.00	103.50	108.00	112.50	117.00	121.50	126.00	130.50	135.00	139.50	144.00
	10	84.00	89.00	94.00	99.00	104.00	109.00	114.00	119.00	124.00	129.00	134.00	139.00	144.00
	11	74.80	80.30	85.80	91.30	96.80	102.30	107.80	113.30	118.80	124.30	129.80	135.30	140.80
	12	62.40	68.40	74.40	80.40	86.40	92.40	98.40	104.40	110.40	116.40	122.40	128.40	134.40

Practice Questions

To help you understand the procedures, we ask that you answer four practice questions using your computer. But here is the good news. For each question you answer correctly, you will earn 10 tokens.

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Prior to the practice round and each of the paid periods, everyone will be randomly assigned into two-firm groups. Because of this randomization, you are very unlikely to be paired with the same firm in more than one decision period.

We will first go through one unpaid practice period. After this practice period, you will be paid based on each and every decision you make.

Before we continue, do you have any questions?

Once you are ready to proceed, please click the “Continue” button.

B.2 Proof for $p^{VB} > p^{NE}$ in Bertrand game.

Given that

$$p^{NE} = \frac{a}{2b - c} \tag{100}$$

and

$$p^{VB} = \frac{a(2b + c)}{2(2b^2 - c^2)} \tag{101}$$

where $a > 0$ and $b \geq c > 0$,

we define

$$\begin{aligned}\frac{p^{VB}}{p^{NE}} &= \frac{\frac{a(2b+c)}{2(2b^2-c^2)}}{\frac{a}{2b-c}} \\ &= \frac{4b^2 - c^2}{4b^2 - 2c^2} > 1\end{aligned} \tag{102}$$

.

Thus, we have showed that

$$p^{VB} > p^{NE} \tag{103}$$

.

C Chapter 3 Appendix

C.1 Proof for independence of complete information/socially optimal effort w.r.t. market state

Proof. Recall the equation that implicitly defines the induced effort under the complete information scenario/social planner case.

$$q_2(\beta, e_E^*) + R_2(\beta_E^*, e) = 0 \tag{104}$$

Differentiating the above equation w.r.t the realized market state, β, we have

$$q_{12}(\beta, e_E^*)d\beta + q_{22}(\beta, e_E^*)de_E^* + R_{12}(\beta, e_E^*)d\beta + R_{22}(\beta, e_E^*)de_E^* = 0 \tag{105}$$

Rearrange the above equation, we have

$$\frac{de_E^*}{d\beta} = -\frac{q_{12}(\beta, e_E^*) + R_{12}(\beta, e_E^*)}{q_{22}(\beta, e_E^*) + R_{22}(\beta, e_E^*)} \tag{106}$$

With the assumptions $q_{12} = 0$ and $R_{12} = 0$, we get that

$$\frac{e_E^*}{d\beta} = 0 \tag{107}$$

The above equation implies that the induced effort under the complete information scenario is independent on the realized market state, β, which is also true for social planner case.

□

C.2 Proof for equivalent simplified version of truth-telling requirement in isolated market

Proof. Since function $e = e(\hat{\beta}, \beta)$ is derived from $\hat{q}(\hat{\beta}) = q(\beta, e)$, then we have

$$e_2(\hat{\beta}, \beta) = -\frac{s_1^{'}(\beta)}{p_1^{'}(e(\hat{\beta}, \beta))} < 0 \tag{108}$$

and

$$e_{12}(\hat{\beta}, \beta) = \frac{s_1^{'}(\beta)p_1^{''}(e(\hat{\beta}, \beta))}{(p_1^{'}(e(\hat{\beta}, \beta)))^2} e_1(\hat{\beta}, \beta) \tag{109}$$

To see the proof of the above assumptions, please see appendix C.3.

Given that $R_{12}(\beta, e) = 0$ and $e_{12}(\hat{\beta}, \beta) = \frac{s_1^{'}(\beta)p_1^{''}(e(\hat{\beta},\beta))}{(p_1^{'}(e(\hat{\beta},\beta)))^2} e_1(\hat{\beta}, \beta)$, the truth-telling requirement (3.12) can be simplified as

$$\int_{\beta'}^{\beta} \int_{\beta'}^{\beta} \{ \frac{s_1^{'}(\beta)[p_1^{'}(e(\hat{\beta}, \beta))p_2^{''}(e(\hat{\beta}, \beta)) - p_2^{'}(e(\hat{\beta}, \beta))p_1^{''}(e(\hat{\beta}, \beta))]}{(p_1^{'}e(\hat{\beta}, \beta))^2} \} e_1(\hat{\beta}, \beta) \geq 0$$

For detailed derivative of the above inequality, please see appendix C.4.

Given the assumption $R_{22}(\beta, e) = -p_2^{''}(e) < 0$, we have $p_2^{''}(e) > 0$. Similarly $q_{22}(\beta, e) = p_1^{''}(e) < 0$. And that $s_1^{'}(\beta) > 0$, $p_1^{'}(e) > 0$, $p_2^{'}(e) > 0$, we have that

$$\frac{s_1^{'}(\beta)[p_1^{'}(e(\hat{\beta}, \beta))p_2^{''}(e(\hat{\beta}, \beta)) - p_2^{'}(e(\hat{\beta}, \beta))p_1^{''}(e(\hat{\beta}, \beta))]}{(p_1^{'}e(\hat{\beta}, \beta))^2} > 0$$

Thus, in order for the condition 3.12 to hold, it is identical to have

$$e_1(\hat{\beta}, \beta) \geq 0$$

Given that $e_1(\hat{\beta}, \beta) = \frac{\hat{q}'(\hat{\beta})}{q_2(\beta, e)}$ and $q_2(\beta, e) > 0$ by assumption, then the above condition is then identical to

$$\hat{q}'(\hat{\beta}) \geq 0$$

So $\hat{q}^{'}(\hat{\beta}) \geq 0$ is the simplified version of the truth-telling requirement.

□

C.3 Derivations of the equations (108) and (109).

Proof. Since function $e = e(\hat{\beta}, \beta)$ is implicitly determined by $\hat{q}(\hat{\beta}) = q(\beta, e) = p_1(\beta) + s_1(e)$, then

$$
\begin{aligned}
e_2(\hat{\beta}, \beta) &= -\frac{q_1(\beta, e)}{q_2(\beta, e)} \\
&= -\frac{{s_1}'(\beta)}{{p_1}'(e)} \\
&= -\frac{s_1^{'}(\beta)}{p_1^{'}(e(\hat{\beta}, \beta))} < 0
\end{aligned}
$$

$$
\begin{aligned}
e_{12}(\hat{\beta}, \beta) &= \frac{\partial^2 e}{\partial \beta \partial \hat{\beta}} \\
&= \frac{d}{d\hat{\beta}}(e_2(\hat{\beta}, \beta)) \\
&= \frac{d}{d\hat{\beta}}[-\frac{s_1^{'}(\beta)}{p_1^{'}(e(\hat{\beta}, \beta))}] \\
&= -s_1^{'}(\beta)(-1){p_1^{'}}^{-2}(e(\hat{\beta}, \beta))p_1^{''}(e(\hat{\beta}, \beta))e_1(\hat{\beta}, \beta) \\
&= \frac{s_1^{'}(\beta)p_1^{''}(e(\hat{\beta}, \beta))}{(p_1^{'}(e(\hat{\beta}, \beta)))^2}e_1(\hat{\beta}, \beta)
\end{aligned}
$$

□

C.4 Simplification of the truth-telling requirement in isolated market case

Proof. Given the assumption $R_{12}(\beta, e) = 0$ and that $e_{12}(\hat{\beta}, \beta) = \frac{s_1^{'}(\beta)p_1^{''}(e(\hat{\beta},\beta))}{(p_1^{'}(e(\hat{\beta},\beta)))^2}e_1(\hat{\beta}, \beta)$, the truth-telling requirement can be simplified as follows.

$$\int_{\beta'}^{\beta}\int_{\beta'}^{\beta} R_{22}(y, e(x,y))e_2(x,y)e_1(x,y) + R_2(y, e(x,y))e_{12}(x,y)dxdy \geq 0$$
$$\int_{\beta'}^{\beta}\int_{\beta'}^{\beta} -p_2^{''}(e(\hat{\beta},\beta))[-\frac{s_1^{'}(\beta)}{p_1^{'}(e(\hat{\beta},\beta))}]e_1(\hat{\beta},\beta) + [-p_2^{'}(e(\hat{\beta},\beta))]\frac{s_1^{'}(\beta)p_1^{''}(e(\hat{\beta},\beta))}{(p_1^{'}(e(\hat{\beta},\beta)))^2}e_1(\hat{\beta},\beta)dxdy \geq 0$$
$$\int_{\beta'}^{\beta}\int_{\beta'}^{\beta}[\frac{s_1^{'}(\beta)p_1^{'}(e(\hat{\beta},\beta))p_2^{''}(e(\hat{\beta},\beta)) - s_1^{'}(\beta)p_2^{'}(e(\hat{\beta},))p_1^{''}(e(\hat{\beta},\beta))}{(p_1^{'}e(\hat{\beta},\beta))^2}]e_1(\hat{\beta},\beta) \geq 0$$
$$\int_{\beta'}^{\beta}\int_{\beta'}^{\beta}\{\frac{s_1^{'}(\beta)[p_1^{'}(e(\hat{\beta},\beta))p_2^{''}(e(\hat{\beta},\beta)) - p_2^{'}(e(\hat{\beta},\beta))p_1^{''}(e(\hat{\beta},\beta))]}{(p_1^{'}e(\hat{\beta},\beta))^2}\}e_1(\hat{\beta},\beta) \geq 0 \tag{110}$$

□

C.5 Proof for sufficiency of the necessary conditions for incentive-compatibility in isolated market case

Proof. In the Isolated market case, the truth-telling requirement, $\hat{q}^{'}(\cdot) \geq 0$, and the first-order condition for truth-telling, $u_{p2}(\beta,\beta) = 0$ is a global optimum for type β. To show this, assume to the contrary that the publisher with market impression quality β strictly prefers to announce $\hat{\beta}$, $\hat{\beta} \neq \beta$.

Then it must be that

$$u_p(\beta,\hat{\beta}) > u_p(\beta,\beta) \tag{111}$$

or equivalently

$$u_p(\beta,\hat{\beta}) - u_p(\beta,\beta) > 0 \tag{112}$$

According to the definite integral rule, the above inequality can be stated as

$$\int_{\beta}^{\hat{\beta}} u_{p2}(\beta,x)dx > 0 \tag{113}$$

where u_{p2} denotes the derivative of the publisher's payoff function w.r.t. the second argument (the announced market impression quality).

Given $u_{p2}(x,x) = 0$ according to the first-order condition for truth-telling, the above inequality is identical to

$$\int_{\beta}^{\hat{\beta}} u_{p2}(\beta, x) - u_{p2}(x, x,)dx > 0 \tag{114}$$

Again, according to the definite integral rule, the above inequality can be stated as

$$\int_{x}^{\beta} \int_{\beta}^{\hat{\beta}} u_{p21}(y, x)dxdy > 0 \tag{115}$$

where u_{p12} always exist by assumption.

Now let's discuss about the sign of the integrand. Recall that

$$\begin{aligned} u_{p12} &= \frac{d}{d\hat{\beta}d\beta}\{(t(\hat{\beta}) + R(\beta, e(\hat{\beta}, \beta)) - R(\beta, 0)\} \\ &= \frac{d}{d\beta}\{t^{'}(\hat{\beta}) + R_2(\beta, e(\hat{\beta}, \beta))e_1(\hat{\beta}, \beta)\} \\ &= (R_{21}(\beta, e(\hat{\beta}, \beta)) + R_{22}(\beta, e(\hat{\beta}, \beta))e_2(\hat{\beta}, \beta))e_1(\hat{\beta}, \beta) + R_2(\beta, e(\hat{\beta}, \beta))e_{12}(\hat{\beta}, \beta) \end{aligned} \tag{116}$$

Given the assumptions $R_{12} = 0$ and $e_{12} = 0$, the above equation reduces to

$$u_{p12} = R_{22}(\beta, e(\hat{\beta}, \beta))e_2(\hat{\beta}, \beta)e_1(\hat{\beta}, \beta) + R_2(\beta, e(\hat{\beta}, \beta))e_{12}(\hat{\beta}, \beta) \tag{117}$$

Given the assumptions that $R_2(\beta, e) < 0$, $R_{22}(\beta, e) < 0$, $e_2(\hat{\beta}, \beta) < 0$, $e_1(\hat{\beta}, \beta) \geq 0$ (the truth-telling requirement), and $e_{12} = \frac{s_1^{'}(\beta)p_1^{''}(e(\hat{\beta},\beta))}{(p_1^{'}(e(\hat{\beta},\beta)))^2}e_1(\hat{\beta}, \beta) < 0$ (as $s_1^{'}(\beta) > 0$, $p_1(e)^{'} > 0$, and $p_1^{''}(e) < 0$), then we can say that

$$u_{p12} \geq 0 \tag{118}$$

[14]

Now let's look at the inequality we have obtained, $\int_{x}^{\beta} \int_{\beta}^{\hat{\beta}} u_{p21}(y, x)dxdy > 0$. If $\hat{\beta} > \beta$, $x > \beta$ for $\forall x \in [\hat{\beta}, \beta]$, the inequality cannot hold. Similarly, if $\hat{\beta} < \beta$, $x < \beta$ for $\forall x \in [\hat{\beta}, \beta]$, we again obtain a contradiction. Now we have proved that the assumption that the publisher

[14] Note that $e_1(\hat{\beta}, \beta) = \frac{\hat{q}^{'}(\hat{\beta})}{q_2(\beta,e)}$, and $q_2(\beta, e) > 0$ by assumption, then $e_1(\hat{\beta}, \beta) \geq 0$ is equivalent to $\hat{q}^{'}(\hat{\beta}) \geq 0$.

with market impression quality β strictly prefers to announce $\hat{\beta} \neq \beta$ can never hold. Thus, the truth-telling requirement $\hat{q}^{'}(\cdot)$ and the first-order condition for truth-telling are sufficient conditions for truth-telling.

□

C.6 Derivation of the publisher's expected rents as a function of β in isolated market case

Proof. The rents function for the publisher with a given realized market situation β can be stated as follows.

$$U_p(\beta) = \int_{\underline{\beta}}^{\beta} p_2^{'}(e(\tilde{\beta}, \tilde{\beta}))[\frac{s_1^{'}(\tilde{\beta})}{p_1^{'}(e(\tilde{\beta}, \tilde{\beta}))}]d\tilde{\beta}$$

By the rule of integrating by parts, we have the expected rents for publisher given the market state distribution, as

$$
\begin{aligned}
\int_{\underline{\beta}}^{\bar{\beta}} U_p(\beta)dF(\beta) &= \int_{\underline{\beta}}^{\bar{\beta}} \int_{\underline{\beta}}^{\beta} R_2(\tilde{\beta}, E(\tilde{\beta}))e_2(\tilde{\beta}, \tilde{\beta})d\tilde{\beta}dF(\beta) \\
&= [F(\beta) \int_{\underline{\beta}}^{\beta} R_2(\tilde{\beta}, E(\tilde{\beta}))e_2(\tilde{\beta}, \tilde{\beta})d\tilde{\beta}]\Big|_{\underline{\beta}}^{\bar{\beta}} \\
&- \int_{\underline{\beta}}^{\bar{\beta}} F(\beta)[R_2(\beta, E(\beta))e_2(\beta, \beta)]d\beta \\
&= \int_{\underline{\beta}}^{\bar{\beta}} R_2(\beta, E(\beta))e_2(\beta, \beta)d\beta \\
&- \int_{\underline{\beta}}^{\bar{\beta}} \frac{F(\beta)}{f(\beta)}[R_2(\beta, E(\beta))e_2(\beta, \beta)]dF(\beta) \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \frac{1}{f(\beta)}[R_2(\beta, E(\beta))e_2(\beta, \beta)]dF(\beta) \\
&- \int_{\underline{\beta}}^{\bar{\beta}} \frac{F(\beta)}{f(\beta)}[R_2(\beta, E(\beta))e_2(\beta, \beta)]dF(\beta) \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \frac{1-F(\beta)}{f(\beta)}[R_2(\beta, E(\beta))e_2(\beta, \beta)]dF(\beta) \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \frac{1-F(\beta)}{f(\beta)}[-p_2^{'}(E(\beta))(-\frac{s_1^{'}(\beta)}{p_1^{'}(E(\beta))})]dF(\beta) \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \frac{1-F(\beta)}{f(\beta)}[p_2^{'}(E(\beta))\frac{s_1^{'}(\beta)}{p_1^{'}(E(\beta))}]dF(\beta)
\end{aligned}
$$

□

C.7 Derivation of the comparative static in isolated market case

Proof. Given the equilibrium effort level equation

$$
p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta)) = \frac{1-F(\beta)}{f(\beta)}[s_1^{'}(\beta)\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p_1^{'}(E^*(\beta)))^2}]
$$

Differentiating the above equilibrium effort level function w.r.t the market situation β, we have

$$p_1^{''}(E^*(\beta))dE^* - p_2^{''}(E^*(\beta))dE^*$$
$$= \{\frac{d}{d\beta}[\frac{1-F(\beta)}{f(\beta)}]s_1^{'}(\beta) + \frac{1-F(\beta)}{f(\beta)}s_1^{''}(\beta)\}[\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p[\frac{p_2^{'}(E^*)}{p_1^{'}(E^*)}]1'(E^*(\beta)))^2}]d\beta$$
$$+ \frac{1-F(\beta)}{f(\beta)}s_1^{'}(\beta)\frac{d}{dE^*}[\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p_1^{'}(E^*(\beta)))^2}]$$
$$p_1^{''}(E^*(\beta))dE^* - p_2^{''}(E^*(\beta))dE^*$$
$$= \{\frac{d}{d\beta}[\frac{1-F(\beta)}{f(\beta)}]s_1^{'}(\beta) + \frac{1-F(\beta)}{f(\beta)}s_1^{''}(\beta)\}[\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p_1^{'}(E^*(\beta)))^2}]d\beta$$
$$+ \frac{1-F(\beta)}{f(\beta)}s_1^{'}(\beta)\frac{d^2}{dE^{*2}}[\frac{p_2^{'}(E^*(\beta))}{p_1^{'}(E^*(\beta))}]dE^*$$

Rearrange the above condition, we find

$$\dot{E}^*(\beta) = \frac{dE^*}{d\beta} = \frac{\frac{d}{d\beta}[\frac{1-F(\beta)}{f(\beta)}]s_1^{'}(\beta) + \frac{1-F(\beta)}{f(\beta)}s_1^{''}(\beta)\}[\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p_1^{'}(E^*(\beta)))^2}]}{p_1^{''}(E^*(\beta)) - p_2^{''}(E^*(\beta)) - \frac{1-F(\beta)}{f(\beta)}s_1^{'}(\beta)\frac{d^2}{dE^{*2}}[\frac{p_2^{'}(E^*(\beta))}{p_1^{'}(E^*(\beta))}]} \tag{119}$$

□

C.8 Proof of the sign of Effort-Market state relationship in isolated market case

Proof. In the model set-up, with the assumption $R_{22}(\beta, e) = -p_2^{''} < 0$, we have $p_2^{''}(e) > 0$. We also have assumptions $p_1^{'}(e) > 0$, $p_2^{'}(e) < 0$, $s_1^{'}(\beta) > 0$, and that $p_1^{''}(e) \geq 0$, $s_1^{''}(\beta) \leq 0$. Then, we have that the terms

$$\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p_1^{'}(E^*(\beta)))^2} > 0 \tag{120}$$

and that

$$p_1^{''}(E^*(\beta)) - p_2^{''}(E^*(\beta)) < 0 \tag{121}$$

Then, with the two assumptions (1) ($\frac{d}{d\beta}\frac{1-F(\beta)}{f(\beta)} < 0$) on the support of $[\underline{\beta}, \bar{\beta}]$), and (2) ($\frac{d^2}{dE^{*2}}[\frac{p_2^{'}(E^*(\beta))}{p_1^{'}(E^*(\beta))}] \geq 0$), we have that

$$\dot{E}^*(\beta) = \frac{dE^*}{d\beta} = \frac{\frac{d}{d\beta}[\frac{1-F(\beta)}{f(\beta)}]s_1^{'}(\beta) + \frac{1-F(\beta)}{f(\beta)}s_1^{''}(\beta)\}[\frac{p_2^{''}(E^*(\beta))p_1^{'}(E^*(\beta)) - p_2^{'}(E^*(\beta))p_1^{''}(E^*(\beta))}{(p_1^{'}(E^*(\beta)))^2}]}{p_1^{''}(E^*(\beta)) - p_2^{''}(E^*(\beta)) - \frac{1-F(\beta)}{f(\beta)}s_1^{'}(\beta)\frac{d^2}{dE^{*2}}[\frac{p_2^{'}(E^*(\beta))}{p_1^{'}(E^*(\beta))}]} > 0$$

□

C.9 Simplification of the truth-telling requirement in correlated markets case

Proof. The truth-telling requirement for any pair of values β and $\beta^{'}$ in $[\underline{\beta}, \bar{\beta}]$ are

$$t(\beta) + \bar{m}(\beta)F_c(\alpha^*|\beta, e(\beta,\beta)) + R(\beta, e(\beta,\beta)) \geq t(\beta') + \bar{m}(\beta')F_c(\alpha^*|\beta, e(\beta',\beta)) + R(\beta, e(\beta',\beta))$$

$$t(\beta') + \bar{m}(\beta')F_c(\alpha^*|\beta', e(\beta',\beta')) + R(\beta', e(\beta',\beta')) \geq t(\beta) + \bar{m}(\beta)F_c(\alpha^*|\beta', e(\beta,\beta')) + R(\beta', e(\beta,\beta'))$$

Adding up the above two inequalities, we have

$$\begin{aligned} &R(\beta, e(\beta,\beta)) - R(\beta, e(\beta',\beta)) + \bar{m}(\beta)F_c(\alpha^*|\beta, e(\beta,\beta)) - \bar{m}(\beta')F_c(\alpha^*|\beta, e(\beta',\beta)) \\ &\geq R(\beta', e(\beta,\beta')) - R(\beta', e(\beta',\beta')) + \bar{m}(\beta)F_c(\alpha^*|\beta', e(\beta,\beta')) - \bar{m}(\beta')F_c(\alpha^*|\beta', e(\beta',\beta')) \end{aligned}$$

Applying the rule of integral over the announced market state argument, we get

$$\begin{aligned} &\int_{\beta'}^{\beta} R_2(\beta, e(x,\beta))e_1(x,\beta) + \bar{m}(x)\frac{d}{de}(F_c(\alpha^*|\beta, e(x,\beta)))e_1(x,\beta) + \frac{d}{x}(\bar{m}(x))F_c(\alpha^*|\beta, e(x,\beta))dx \\ &\geq \int_{\beta'}^{\beta} R_2(\beta', e(x,\beta'))e_1(x,\beta') + \bar{m}(x)\frac{d}{de}(F_c(\alpha^*|\beta', e(x,\beta')))e_1(x,\beta') + \frac{d}{x}(\bar{m}(x))F_c(\alpha^*|\beta', e(x,\beta'))dx \end{aligned}$$

Given $R_{12} = 0$, and applying the integration rule w.r.t. the true market state argument, the above inequality is identical to

$$\int_{\beta'}^{\beta}\int_{\beta'}^{\beta} R_{22}(y, e(x, y))e_2(x, y)e_1(x, y) + R_2(y, e(x, y))e_{12}(x, y)$$
$$+ \bar{m}(x)e_1(x, y)[\frac{d}{ded\beta}(F_c(\alpha^*|y, e(x, y)) + \frac{d}{de^2}F_c(\alpha^*|y, e(x, y))e_2(x, y)]$$
$$+ \frac{d}{x}(\bar{m}(x))[\frac{d}{de}(F_c(\alpha^*|y, e(x, y)) + \frac{d}{de}F_c(\alpha^*|y, e(x, y))e_2(x, y)]dxdy \geq 0$$

or equivalently

$$\int_{\beta'}^{\beta}\int_{\beta'}^{\beta} R_{22}(y, e(x, y))e_2(x, y)e_1(x, y) + R_2(y, e(x, y))e_{12}(x, y)$$
$$+ \frac{d}{dx}[\bar{m}(x)[\frac{d}{d\beta}F_c(\alpha^*|y, e(x, y)) + \frac{d}{de}F_c(\alpha^*|y, e(x, y))e_2(x, y)]]dxdy \geq 0$$

In order for the above inequality to hold, we need to have

$$R_{22}(y, e(x, y))e_2(x, y)e_1(x, y) + R_2(y, e(x, y))e_{12}(x, y)$$
$$+ \frac{d}{dx}[\bar{m}(x)[\frac{d}{d\beta}F_c(\alpha^*|y, e(x, y)) + \frac{d}{de}F_c(\alpha^*|y, e(x, y))e_2(x, y)]] \geq 0$$

The above inequality is the simplified condition which is equivalent to the truth-telling requirement in the isolated market case.

□

C.10 Proof for sufficiency of the necessary conditions for incentive-compatibility in correlated markets case

Proof. In the model, the truth-telling requirement $R_{22}(y, e(x, y))e_2(x, y)e_1(x, y) + \frac{d}{dx}[\bar{m}(x)[\frac{d}{d\beta}F_c(\alpha^*|y, e(x, y)$ $\frac{d}{de}F_c(\alpha^*|y, e(x, y))e_2(x, y)]] \geq 0$, and the first-order condition $u_{p2}(\beta, \beta) = 0$ imply that $\hat{\beta} = \beta$

is a global optimum for type β. To show this, assume to the contrary that type β strictly prefers to announce $\hat{\beta}$, where $\hat{\beta} \neq \beta$.

Then it must be that

$$u_p(\beta, \hat{\beta}) > u_p(\beta, \beta) \tag{122}$$

or equivalently

$$u_p(\beta, \hat{\beta}) - u_p(\beta, \beta) > 0 \tag{123}$$

According to the definite integral rule, the above inequality can be state as

$$\int_{\beta}^{\hat{\beta}} u_{p2}(\beta, x)dx > 0 \tag{124}$$

Given that $u_{p2}(\beta, \beta) = 0$ by the first-order condition for truth-telling, we have get the following inequality

$$\int_{\beta}^{\hat{\beta}} u_{p2}(\beta, x) - u_{p2}(x, x)dx > 0 \tag{125}$$

Again by the definite integral rule, we can state the above inequality as below

$$\int_{x}^{\beta} \int_{\beta}^{\hat{\beta}} u_{p21}(y, x)dxdy > 0 \tag{126}$$

where u_{p12} exits by assumption and satisfies the following condition

$$\begin{aligned} u_{p12}(y, x) = {} & R_{22}(y, e(x, y))e_2(x, y)e_1(x, y) + \\ & \frac{d}{dx}[\bar{m}(x)[\frac{d}{d\beta}F_c(\alpha^*|y, e(x, y)) + \frac{d}{de}F_c(\alpha^*|y, e(x, y))e_2(x, y)]]dxdy \geq 0 \end{aligned} \tag{127}$$

by the truth-telling requirement.

Then we can see that if $\hat{\beta} > \beta$, $x \geq \beta$ for $\forall x \in [\beta, \hat{\beta}]$, and the inequality cannot hold. Similarly, if $\hat{\beta} < \beta$. $x \leq \beta$ for $\forall x \in [\hat{\beta}, \beta]$, and again we obtain a contradiction. Now we have proved that the assumption that the publisher with market impression quality β

strictly prefers to announce $\hat{\beta} \neq \beta$ can never hold. Thus the truth-telling requirement and the first-order condition are sufficient conditions for truth-telling.

□

C.11 Marginal rents function for the publisher under truth-telling conditions in correlated markets case

Proof. Let $U_p(\beta, \alpha) \equiv u_p(\beta, \hat{\beta} = \beta, \alpha)$ denotes the publisher's rent when the true market impression quality is β under the truth-telling requirement. We have

$$U_p(\beta, \alpha) \equiv t(\beta) + \bar{m}(\beta) F_c(\alpha^* | \beta, e(\beta, \beta)) + R(\beta, e(\beta, \beta)) - R(\beta, 0) \tag{128}$$

Then, the envelope theorem applied to the maximization of the publisher's objective function 3.29) yields

$$\begin{aligned}
\dot{U}_p(\beta) &= \bar{m}(\beta)[\frac{d}{d\beta}(F_c(\alpha^*|\beta, e(\beta,\beta)) + \frac{d}{de}(F_c(\alpha^*|\beta, e(\beta,\beta))e_2(\beta,\beta))] \\
&+ R_1(\beta, e(\beta,\beta)) + R_2(\beta, e(\beta,\beta))e_2(\beta,\beta) - R_1(\beta, 0) \\
&= \bar{m}(\beta)[\frac{d}{d\beta}(F_c(\alpha^*|\beta, e(\beta,\beta)) + \frac{d}{de}(F_c(\alpha^*|\beta, e(\beta,\beta))e_2(\beta,\beta))] + R_2(\beta, e(\beta,\beta))e_2(\beta,\beta)
\end{aligned} \tag{129}$$

where $\dot{U}_p(\beta)$ denotes the derivative of the publisher's objective function w.r.t. the true market state β under truth-telling condition. We name it the publisher's "rent speed" function w.r.t. β. Recall the reward magnitude function (3.36),

$$\bar{m}(\beta) = -\frac{R_2(\beta, e(\beta,\beta))e_2(\beta,\beta) - \phi(\beta)}{\frac{d}{d\beta}(F_c(\alpha^*|\beta, e(\beta,\beta))) + \frac{d}{de}(F_c(\alpha^*|\beta, e(\beta,\beta))e_2(\beta,\beta))} \tag{130}$$

Rearrange the above function, we have

$$\bar{m}(\beta)[\frac{d}{d\beta}(F_c(\alpha^*|\beta, e(\beta,\beta))) + \frac{d}{de}(F_c(\alpha^*|\beta, e(\beta,\beta))e_2(\beta,\beta))] + R_2(\beta, e(\beta,\beta))e_2(\beta,\beta)$$
$$= \phi(\beta) \tag{131}$$

By combining the above two equations, it is obvious to see that $\dot{U}(\beta) = \phi(\beta)$.

□

C.12 Proof for the simplification of truth-telling condition in correlated markets case

Recall the Eq (131),

$$\bar{m}(\beta)[\frac{d}{d\beta}(F_c(\alpha^*|\beta, e(\beta,\beta))) + \frac{d}{de}(F_c(\alpha^*|\beta, e(\beta,\beta))e_2(\beta,\beta))] + R_2(\beta, e(\beta,\beta))e_2(\beta,\beta)$$
$$= \phi(\beta)$$

Differentiating the above equation w.r.t to the announced market state argument, we have (To make it clear to see, we use y to denote the true market state argument and x to denote the announced market state argument)

$$\frac{d}{dx}\{\bar{m}(x)[\frac{d}{d\beta}F_c(\alpha^*|y, e(x,y)) + \frac{d}{de}F_c(\alpha^*|y, e(x,y))e_2(x,y)]\}$$
$$R_{22}(y, e(x,y))e_1(x,y)e_2(x,y) + R_2(y, e(x,y))e_{21}(x,y)$$
$$= \frac{d}{x}\phi(x)$$

Recall the reduced truth-telling requirement 3.34, the left-hand side of the above equation coincident with the left-hand side of the truth-telling requirement. We can tell that

$$\frac{d}{dx}\phi(x) \geq 0 \tag{132}$$

is identical to the truth-telling requirement (3.34).

C.13 Proof for the guaranteed payment in numerical example correlated case

Proof. According to the guaranteed payment function developed in the theoretical section, and the spot market revenue $R(\beta, e)$ function set up in the numerical example section, we have

$$
\begin{aligned}
t(\hat{\beta}) &= -\bar{m}(\hat{\beta})F_c(\alpha^*|\hat{\beta}, E^*(\hat{\beta})) - R(\hat{\beta}, E^*(\hat{\beta})) + R(\hat{\beta}, 0) \\
&= -\bar{m}(\hat{\beta})F_c(\alpha^*|\hat{\beta}, \Theta(\hat{\beta})) - (\hat{\beta} - \Theta(\hat{\beta})^2) + \hat{\beta} \\
&= -\bar{m}(\hat{\beta})F_c(\alpha^*|\hat{\beta}, \Theta(\hat{\beta})) + \Theta(\hat{\beta})^2 \\
&= \frac{4\Theta(\hat{\beta})^{\frac{3}{2}}}{\frac{d}{d\beta}(F_c(\alpha^*|\beta, \Theta(\hat{\beta}))) - \frac{d}{de}(F_c(\alpha^*|\beta, \Theta(\hat{\beta}))(2\Theta(\hat{\beta})^{\frac{1}{2}}} F_c(\alpha^*|\hat{\beta}, \Theta(\hat{\beta})) + \Theta(\hat{\beta})^2 \\
&= \frac{1.0119}{\frac{d}{d\beta}(F_c(\alpha^*|\beta, 0.4)) - 1.2649\frac{d}{de}(F_c(\alpha^*|\beta, 0.4)} F_c(\alpha^*|\hat{\beta}, 0.4) + 0.4^2 \\
&= \frac{1.0119}{2.0119\frac{d}{dx}(F_c(\alpha^*|\hat{\beta}, 0.4))} F_c(\alpha^*|\hat{\beta}, 0.4) + 0.16
\end{aligned}
\tag{133}
$$

where $\Theta(\hat{\beta}) = 0.4$.

□

C.14 Proof for the reward magnitude in numerical example correlated case

Proof. According to the contracting mechanism for the correlated market developed in the section (3.4), if the realized spot market aggregated price is smaller the given threshold α^*, the traditional contractor would set the reward $\bar{m}(\hat{\beta})$ for the publisher as

$$
\begin{aligned}
\bar{m}(\hat{\beta}) &= -\frac{R_2(\hat{\beta}, \Theta(\hat{\beta}))e_2(\hat{\beta}, \hat{\beta}) - \phi(\hat{\beta})}{\frac{d}{d\hat{\beta}}(F_c(\alpha^*|\hat{\beta}, \Theta(\hat{\beta}))) + \frac{d}{de}(F_c(\alpha^*|\hat{\beta}, \Theta(\hat{\beta}))e_2(\hat{\beta}, \hat{\beta}))} \\
&= -\frac{1.0119 - \phi(\hat{\beta})}{\frac{d}{d\hat{\beta}}(F_c(\alpha^*|\hat{\beta}, 0.4)) - 1.2649\frac{d}{de}(F_c(\alpha^*|\hat{\beta}, 0.4)}
\end{aligned}
\tag{134}
$$

since $\Theta(\hat{\beta}) = 0.4$.

Let's use $\frac{d}{dx}(F_c(\alpha^*|x))$ to denote the derivative of α's cumulative distribution function at the point of α^* w.r.t. the mean of the distribution. Given that α's mean is determined by $\beta - e^2$. Then we have that $\frac{d}{d\beta}F_c(\alpha^*|\beta - e^2) = \frac{d}{dx}F_c(\alpha^*|x)$ and $\frac{d}{de}F_c(\alpha^*|\beta - e^2 = -2e\frac{d}{dx}F_c(\alpha^*|x)$. What's more, since the incentive-compatible condition for the publisher is $\frac{d}{d\hat{\beta}}\phi(\hat{\beta}) \geq 0$ and $\phi(\beta)$ denotes the publisher's rents under the incentive-compatible condition, then it is the traditional advertiser's best interest to set $\phi(\hat{\beta})$ equal to zero (or approximate zero). Then the magnitude of the reward reduces to

$$\begin{aligned}\bar{m}(\hat{\beta}) &= -\frac{1.0119}{\frac{d}{dx}(F_c(\alpha^*|\hat{\beta}, 0.4)) + 1.2649 \times 2 \times 0.4 \times \frac{d}{dx}(F_c(\alpha^*|\hat{\beta}, 0.4)} \\ &= -\frac{1.0119}{2.0119\frac{d}{dx}(F_c(\alpha^*|\hat{\beta} - 0.16))}\end{aligned} \tag{135}$$

as $F_c(\alpha^*|\beta, e) = F_c(\alpha^*|\beta - e^2)$.

□

Vita

Na Zuo was born on the fourth day of February 1990 in Chongqing, China. She had a great childhood living in Chongqing until 1997 and then in Chengdu, China, until She moved to Shanghai to attend the East China University of Science and Technology in September of 2008. She received a Bachelor of Science in Finance, from the East China University of Science and Technology, June of 2012. She then moved to Urbana, Illinois and earned a Master of Science degree in Economics from the University of Illinois at Urbana-Champaign, December of 2013. In August of 2014, she has started her Ph.D. study at the University of Tennessee, Knoxville, and anticipate to graduate in August of 2020. She has accepted the offer from Shandong University at the position of Assistant Professor in Jinan, China.